Working Paper Series (E)

No.20

Why Do Age-Specific Marital Fertility Rates Fail?

Toru Suzuki

February 2004

National Institute of Population and Social Security Research

2-2-3 Uchisaiwai-cho, Chiyoda-ku,
Tokyo, Japan

No.20

Why Do Age-Specific Marital Fertility Rates Fail?

Toru Suzuki

February 2004

Why Do Age-Specific Marital Fertility Rates Fail?

Toru Suzuki
E-mail: suzuki-t@ipss.go.jp

1. Introduction

Decomposition analyses using the age-specific marital fertility rates (AMFRs) have been frequently conducted both in Japan (阿藤, 1992, p. 51; 河野, 1995, pp. 67-71; Tsuya and Mason, 1995, pp. 147-148; 国立社会保障・人口問題研究所, 1997, p. 10) and in Korea (김승권· 외, 2002, p. 77; 전광희, 2002, pp. 90-94; Eun, 2003, p. 582). These studies asserted that recent fertility decline was mostly explained by nuptiality decline and that marital fertility did not play an important role. However, Hirosima (廣嶋, 2001) showed that the AMFRs are erroneous when there is a trend of marriage postponement. This paper attempts to clarify the problem of AMFRs not with a simulation model like Hirosima's but with an analytical model. While Hirosima assumed that the marital fertility is independent of the age at marriage, this paper explores more general results.

2. Framework

An analytical framework for marriage and fertility behaviors derives from Inaba (1995). Let x be the current age, a be the age at first marriage, and y be the marital duration. Let a and ß be the lower and upper limit of childbearing age. It is assumed that there is no divorce, remarriage or death during the childbearing age. Thus, the term "marriage" always means first marriage and there is no marital status other than "single" and "(currently) married". The ordinary age-specific fertility rate $f(x)$ can be expressed as follows.

$$f(x) = \int_{a}^{x} \phi(a)\, m(a, x-a)\, da, \qquad (2\text{-}1)$$

where, $f(a)$ age specific first marriage rates,
$m(a,y)$ fertility rates by marriage age and marital duration.

Two cumulative functions are defined. While $\Phi(x)$ is the proportion of married female by age, $M(a)$ is the average number of children eventually born by age at marriage.

$$\Phi(x) = \int_{\alpha}^{x} \phi(a)\,da. \tag{2-2}$$

$$M(a) = \int_{\alpha}^{\beta-a} m(a, y)\,dy. \tag{2-3}$$

All these functions are supposed to express the marriage and childbearing behavior of a hypothetical cohort in a specific year. The total fertility rate can be expressed as follows.

$$TFR = \int_{\alpha}^{\beta} f(x)\,dx = \int_{\alpha}^{\beta} \phi(a) M(a)\,da. \tag{2-4}$$

The following assumptions are made. Assumption 1 may not require any justification. Assumption 2 is a simplification of the reality ignoring a possible increase immediately after the marriage. Assumption 3 is fairly supported at least in Japan. A brief inspection of vital statistics reveals that there are very few cases that violate the assumption.

Assumption 1. $\phi(a)$ has a unimodal age pattern.
Assumption 2. $m(a,y)$ is monotonously decreasing both in terms of a and y.
Assumption 3. $\phi(a)/\Phi(a)$ is monotonously decreasing in terms of a.

3. Decomposition Method

The AMFR at age x is defined as follows.

$$AMFR(x) = \frac{f(x)}{\Phi(x)}. \tag{3-1}$$

Thus, $f(x)$ can be seen as a product of AMFR and the proportion married at age x. For such an index, Kitagawa (1955) proposed a decomposition method that does not produce residual. In the following, ΔTFR is the total change, ΔTFR_n is the change due to the marriage behavior, and ΔTFR_f is that due to the childbearing behavior.

$$\Delta TFR = TFR_2 - TFR_1 = \int_{\alpha}^{\beta+h} \{f_2(x) - f_1(x)\}\,dx, \tag{3-2a}$$

$$\Delta TFR_n = \int_{\alpha}^{\beta+h} \{\Phi_2(x) - \Phi_1(x)\} \cdot \frac{1}{2}\left\{\frac{f_2(x)}{\Phi_2(x)} + \frac{f_1(x)}{\Phi_1(x)}\right\} dx, \tag{3-2b}$$

$$\Delta TFR_f = \int_{\alpha}^{\beta+h} \left\{ \frac{f_2(x)}{\Phi_2(x)} - \frac{f_1(x)}{\Phi_1(x)} \right\} \cdot \frac{1}{2} \{\Phi_2(x) + \Phi_1(x)\} dx. \tag{3-2c}$$

Table 1 shows one of Hirosima's simulations. He assumed that the marital fertility is independent of the age at marriage and that $m(1) = 1$, $m(2) = 1$, and $m(y) = 0$ for $y > 2$. Thus, $f(x) = f(x-1) + f(x-2)$ in this case. Marriage postponement is simulated such that $\phi_2(x) = \phi_1(x-1)$. Although $m(y)$ is held constant and $\Delta TFR = 0$, the decomposition suggests that marital fertility affected in the direction to raise the TFR while nuptiality affected in the opposite direction. Thus, the decomposition is apparently problematic in this scenario.

Table 1. Decomposition of TFR change in marriage postponement

x	$f_1(x)$	$F_1(x)$	$f_1(x)$	$f_1(x)/F_1(x)$	$f_2(x)$	$F_2(x)$	$f_2(x)$	$f_2(x)/F_2(x)$	$\Delta(x)$	$\Delta_n(x)$	$\Delta_f(x)$
15	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
16	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
17	0.10	0.05	0.05	1.00	0.05	0.00	0.00	0.00	-0.05	-0.03	-0.03
18	0.15	0.15	0.15	1.00	0.10	0.05	0.05	1.00	-0.10	-0.10	0.00
19	0.20	0.30	0.25	0.83	0.15	0.15	0.15	1.00	-0.10	-0.14	0.04
20	0.20	0.50	0.35	0.70	0.20	0.30	0.25	0.83	-0.10	-0.15	0.05
21	0.15	0.70	0.40	0.57	0.20	0.50	0.35	0.70	-0.05	-0.13	0.08
22	0.10	0.85	0.35	0.41	0.15	0.70	0.40	0.57	0.05	-0.07	0.12
23	0.05	0.95	0.25	0.26	0.10	0.85	0.35	0.41	0.10	-0.03	0.13
24	0.00	1.00	0.15	0.15	0.05	0.95	0.25	0.26	0.10	-0.01	0.11
25	0.00	1.00	0.05	0.05	0.00	1.00	0.15	0.15	0.10	0.00	0.10
26	0.00	1.00	0.00	0.00	0.00	1.00	0.05	0.05	0.05	0.00	0.05
Total			2.00				2.00		0.00	-0.66	0.66

Source: Hirosima (2001), p. 177.

4. Analytical Approach to Decomposition Failure

Since Hirosima's purpose was to show that the decomposition using the AMFRs can fail when there is a trend of marriage postponement, it was sufficient for him to give one specific case of failure. However, more general result is explored here employing an analytical approach.

First, it can be shown that the result of no change in TFR depends on the assumption of independence between the age at marriage and marital fertility. Changing the order of integration of (2-1) and applying (2-3), the new TFR is expressed as follows.

$$TFR_2 = \int_{\alpha}^{\beta+h} \phi(a-h) M(a) da. \tag{4-1}$$

If the marriage timing and marital fertility are independent each other, $M(a)$ is constant M and TFR_2 = TFR_1. In a more general situation, $M(a)$ is monotonously declining by the Assumption 2. Because $M(a)$ is larger where $\phi(a\text{-}h)$ is smaller than $\phi(a)$, TFR_2 must be smaller than TFR_1. Thus, a delay in marriage causes a decline in TFR.

It also can be shown that ΔTFR_f is always greater than ΔTFR. Expanding (3-2c), substituting the definition of $f_1(x)$ and $f_2(a)$, and changing the order of integration give the following expression.

$$\Delta TFR_f = \frac{1}{2}\Delta TFR + \frac{1}{2}\left\{\int_a^{\beta+h} \phi(a-h)\ M^+(a)\,da - \int_a^{\beta+h} \phi(a)\ M^-(a)\,da\right\}, \quad (4\text{-}2a)$$

$$\text{where,}\quad M^+(a) = \int_a^{\beta+h} \frac{\Phi(x)}{\Phi(x-h)} m(a, x-a)\,dx, \quad (4\text{-}2b)$$

$$M^-(a) = \int_a^{\beta+h} \frac{\Phi(x-h)}{\Phi(x)} m(a, x-a)\,dx. \quad (4\text{-}2c)$$

Because of the relation that $M^-(a) < M(a) < M^+(a)$, the second term of (4-2a) is larger than $\Delta TFR/2$. Although the correct relations should be $\Delta TFR_f = 0$ and $\Delta TFR_n = \Delta TFR$, the decomposition analysis results in a wrong relation that $\Delta TFR_f > \Delta TFR$.

5. Serious Failure

If ΔTFR_f is negative, ΔTFR_n is also negative. This situation produces a reasonable interpretation that both marriage timing and marital fertility contributed to a decline in TFR. However, if ΔTFR_f is positive, a radical interpretation arises that while marital fertility affected to raise TFR, marriage timing overcame it to make TFR decline. Let us call such a situation "serious failure".

To determine the condition for the serious failure to happen, the condition for each AMFR to rise is explored. For a given age x and delay in marriage h, the following function is defined.

$$H(a) = \frac{\phi(a-h)}{\Phi(x-h)} - \frac{\phi(a)}{\Phi(x)}, \quad a \le x. \quad (5\text{-}1)$$

The change in AMFR is expressed with $H(a)$ as follows.

$$\frac{f_2(x)}{\Phi_2(x)} - \frac{f_1(x)}{\Phi_1(x)} = \int_a^x H(a) m(a, x-a)\,da. \quad (5\text{-}2)$$

Note that a non-weighted integration of $H(a)$ is 0. It is apparent that $H(a) < 0$ in early ages. By the Assumption 3, $H(a) > 0$ as a approaches to x. Thus, the AMFR increases if $m(a,x-a)$ is increasing in terms of a. This happens if the slope of $m(a,y)$ is steeper in terms of marital duration (y) than that in terms of marriage timing (a).

Thus, the serious failure is not mathematically inevitable but depends on the pattern of $m(a,x-a)$. However, it can be shown that the actual pattern of $m(a,y)$ is favorable to the serious failure. Suppose oppositely that $m(a,x-a)$ is decreasing in terms of a. In such a case, a rise in age-specific fertility rate is never observed.

$$f_2(x) - f_1(x) = \int^x \{\phi(a-h) - \phi(a)\} m(a, x-a)\, da < 0. \qquad (5\text{-}3)$$

In the real world, however, recuperations of fertility in older ages are frequently observed. This fact suggests that $m(a,x-a)$ is actually increasing in terms of a and tends to cause the serious failure.

As a direct evidence in Japan, Figure 1 shows the pattern of $m(a,x-a)$ estimated from The Fourth National Survey on Household Changes conducted by The National Institute of Population and Social Security Research in 1999. Although there are some up and down, $m(a,x-a)$ is basically increasing.

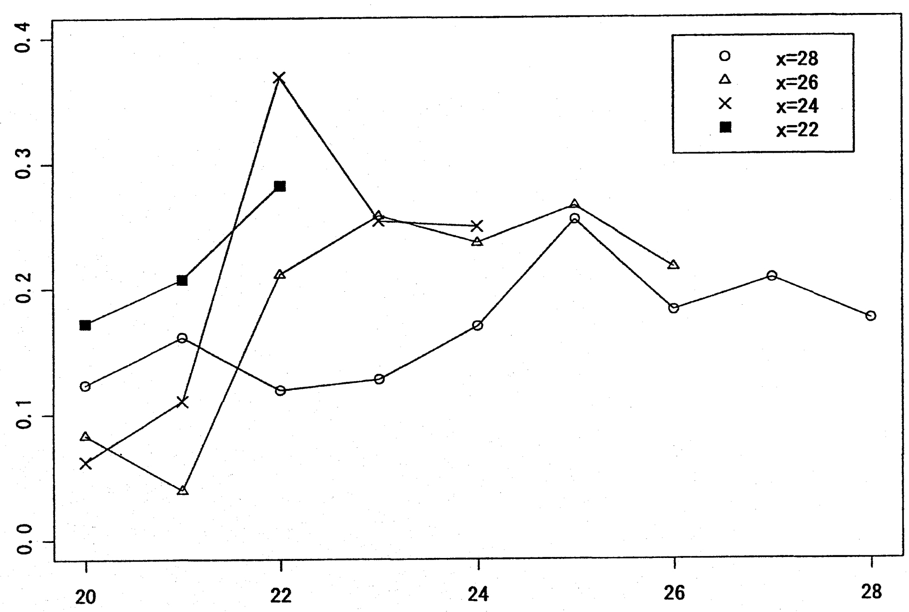

Figure 1. Pattern of $m(a,y)$ in recent Japan.
Source: The Fourth National Survey on Household Changes, 1999.

6. Deceptive Increase

This section investigates the condition for the AMFR to rise when the genuine marital fertility is declining. Assume that there was a uniform change in marital fertility that $m(a,y) \rightarrow c\ m(a,y)$. Also assume that $m(a,x\text{-}a)$ at given x can be approximated by a straight line.

$$m(a, x-a) = u(x) + v(x)\,a. \qquad (6\text{-}1)$$

The change in the AMFR can be expressed as follows.

$$\frac{f_2(x)}{\Phi(x-h)} - \frac{f_1(x)}{\Phi(x)} = u(x)(c-1) + v(x)\big[c\{\mu(x-h)+h\} - \mu(x)\big] \qquad (6\text{-}2)$$

Here, $\mu(x)$ is the mean age at marriage for those who married by age x.

$$\mu(x) = \frac{1}{\Phi(x)} \int^{x} a\,\phi(a)\,da. \qquad (6\text{-}3)$$

The condition for the AMFR to rise is as follows.

$$c > \frac{\mu(x) + u(x)/v(x)}{\mu(x-h) + h + u(x)/v(x)}. \qquad (6\text{-}4)$$

Because $\mu(x\text{-}h) + h$ is usually greater than $\mu(x)$, it is possible that c is less than unity. This means that the AMFR can rise when the genuine marital fertility is declining.

7. Summary

Following results were obtained under the situation of marriage postponement.

1. TFR decreases even though marital fertility is held constant.
2. The ordinary decomposition tends to underestimate the effect of marital fertility.
3. The actual pattern of marital fertility tends to cause the serious failure of decomposition analysis.
4. AMFR can rise while the genuine marital fertility is declining.

References

阿藤誠 (1992),「日本における出生率の動向と要因」, 河野稠果・岡田實(編)『低出生力をめぐる諸問題』, 大明堂, pp. 48-68.

전광희 (2002), "출산력", 김두섭・박상태・은기수 편 『한국의 인구』 통계청, pp. 81-113.

Eun, Ki-Soo (2003), "Understanding recent fertility decline in Korea," *Journal of Population and Social Security*, Supplement to Volume 1, http://www.ipss.go.jp/English/WebJournal.files/Population/2003_6/20.Eun.pdf

廣嶋清志 (2001),「出生率低下をどのようにとらえるか? —年齢別有配偶出生率の問題性—」『理論と方法』第 16 巻第 2 号, pp. 163-183.

Inaba, H. (1996), "Human population reproduction via first marriage," *Mathematical Population Studies*, 5, pp. 123-144.

김승권· 최변호· 정경화 이삼식· 박덕규· 박인화· 장지연 (2002),『低出産의 社會經濟的 影響과 長・短期政策方案』韓國保健社會研究院.

Kitagawa, E. M. (1955), "Components of a difference between two rates," *Journal of American Statistical Association*, 50, pp.1168-1194..

河野稠果 (1995),「配偶関係と出生力」日本統計協会(編)『現代日本の人口問題』大蔵省印刷局, pp. 63-110.

国立社会保障・人口問題研究所 (1997),『日本の将来推計人口：平成 9 年 1 月推計』研究資料第 291 号.

Tsuya, Noriko O. and Karen Oppenheim Mason (1995), "Changing gender roles and below-replacement fertility in Japan," in Karen Oppenheim Mason and AnMagritt Jensen (eds.), *Gender and Family Change in Industrial Countries*, Oxford, Clarendon Press, pp. 139-167.

Working Paper Series(E)

＊Working Paper Series are provided for discussions and critical comments, and their circulation is limited.

Do not quote and cite without a written consent from the authors.

1 The Pattern of Intergenerational Mobility Among Men and Woman
(Sawako Shirahase)
July 1997

2 Pigou' s Welfare Economics and Ethical Belief
(Yuichi Shionoya)
February 1998

3 Health expenditure of deceased elderly in Japan
(Tetsuo Fukawa)
February 1998

4 Population Development and Social Problems in the Inner City and Suburbs of the Tokyo Metropolitan Area
(Satoshi Nakagawa)
April 1998

5 A Social Procedure for Choosing Fair Allocation Rules
-Formalizing the Rawlsian Principles of Justice-
(Reiko Gotoh and Naoki Yoshihara)
April 1998

6 Economy and Morality:A Conceptual Framework
(Yuichi Shionoya)
May 1998

7 Schumpeter on the Relationship between Economics and Sociology from the Perspective of Doctrinal History
(Yuichi Shionoya)
May 1998

8 Women and the Class Structure in Japan and Great Britain
(Sawako Shirahase)
March 1999

9 The Transformation of Partnerships of Japanese Women in the 1990s:
(Miho Iwasawa)
March 2000

10 Effects of Ethnic Concentration on migration in peninsular Malaysia
(Yoshimi Chitose)
June 2000

11 Japanese Income Inequality by Household Types in Comparative Persepective
(Sawako Shirahase)
September 2001

12 Effects of Government Policy on Internal Migration in Peninsular Malaysia: A Comparison between Malays and Non-Malays
(Yoshimi Chitose)
March 2002

13 What Determines Earning Differentials in Urban China?
(Yanfei Zhou)
April 2002

14 Women' s Working Pattern and the Support to Working Mothers in Contemporary Japan
(Sawako Shirahase)
September 2002

15 Leaving Home in Japan: Its Trends, Gender Differences, and Determinants
(Toru Suzuki)
November 2002

16 Elaboration of the Coale-McNeil Nuptiality Model as
The Generalized Log Gamma Distribution:
New Identity and Empirical Enhancement

(Ryuichi Kaneko)

December 2002

17 Transformations in First Marriage Behavior of Japanese Female Cohorts:
Estimation and Projection of Lifetime Measures via
Empirically Adjusted Coale-McNeil Model

(Ryuichi Kaneko)

December 2002

18 Three Worlds of Pronatalism?:
Fertility and Family-Friendly Policies in Europe and Japan

(Nobutaka Fukuda)

January 2003

19 "Have we really reached the western standard of retired income security?
- From the viewpoints of public and private pension schemes- "

(Yukiko M. Katsumata)

May 2003

20 Why Do Age-Specific Marital Fertility Rates Fail?

(Toru Suzuki)

February 2004