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A Social Procedure for Choosing Fair Allocation Rules
— Formalizing the Rawlsian Principles of Justice —

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Abstract: In this paper, we consider and formulate, in cooperative production economies, a *social procedure for choosing fair allocation rules*, basing it upon the argument of the *Rawlsian two principles of justice* (Rawls (1971)). First, we adopt Sen's *Capability* (Sen (1980, 1985)) index to evaluate individual states, instead of income or welfare. Second, we define individual i's *judgement* over allocation rules as i's proposing social welfare function, which assigns to each economic environment an ordering over ordered pairs of allocation rules and feasible allocations. Third, we formalize a *social decision procedure of fair allocation rules* (**SDPR**) as a function which aggregates a profile of individual judgements over ordered pairs of allocation rules and feasible allocations into a social judgement. We also define several conditions on individual judgements, each of which seems to be a requirement consistent with the Rawlsian two principles of justice. An **SDPR** is defined as the *Rawlsian* one (**RSDPR**) if its domain is restricted to the one which meets these conditions, and it satisfies the *Pareto Principle.* We characterize the class of allocation rules, each of which is selected in the corresponding economic environment through the **RSDPR**. We also show that under some assumptions, these selected rules have non-empty sets of feasible allocations which are consistent with the purpose of the Rawlsian difference principle. Finally, we discuss the "moral hazard" problem inherent in the **RSDPR**.

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1 Introduction

One of the main purposes of the traditional social choice theory in economic environments is to propose and/or characterize "desirable" allocation rules (social choice correspondences), of which ranges are the set of feasible allocations, and of which domains include the class of possible profiles of all individuals' preference orderings. Then, the desirability of each allocation rule is represented either by a group of axioms which characterize this rule, or by an intuitive concept of rule itself. Note that there are usually many criteria for "desirability" of allocation rules. For instance, in the theory of equitable allocations, there are various fair allocation rules proposed by various researchers, like the no-envy (Foley (1967)), egalitarian-equivalent (Pazner & Schmeidler (1978)), and wealth-fair rules (Varian (1974)). We can, then, consider a scenario that in a democratic society, one "equitable" allocation rule is selected among various "equitable" rules through a social choice procedure. Such a social procedure can guarantee individuals in the society the rights to participate in the process of choosing allocation rules, so that they commit themselves to follow the rule which they choose.

According to the discussion of Rawls (1971), we can formulate a social procedure for choosing one allocation rule among various ones. Rawls (1971) proposed *two principles of justice* which are the *higher order principles* to stipulate what types of allocation rules should be admitted as *fair* ones. The *first principle of justice* by Rawls (1971) is the *equal basic liberties* which requires *fair* allocation rules to guarantee every individual at least a formal right of freedom of choice and action. The *second principle of justice* consists of the *fair equality of opportunity* and the *difference principle*, the latter of which stipulates *fair* allocation rules by balancing, from the viewpoint of "the greatest expectation of *the least advantaged*," various different "*precepts*" (Rawls (1971)) on what is distributive justice. These principles refer to not only what allocations are fair, but also what procedural characteristics fair allocation rules should have. By adopting these arguments, we can consider, with regard to a social procedure for choosing one among various candidates of "fair" allocation rules, the following scenario: a society can eliminate sets of allocation rules which are inadequate from the viewpoint of the Rawlsian two principles of justice, if it accepts these principles. Moreover, if there still remain multiple "fair" allocation rules which pass the test of the Rawlsian principles, then the society may select one among them by aggregating individuals' judgements about which of the "fair" allocation rules is just into a social judgement.

In this paper, we consider and formulate, in cooperative production economies, a *social decision procedure of fair allocation rules*, basing it upon the argument of the *Rawlsian principles of justice* (Rawls (1971)). The *allocation rules* discussed in this paper are defined as *social choice correspondences* or *game forms.* Among various "precepts" of distributive justice, in this paper, we adopt two of them in particular —"*distribution with regard to individual contributions*" and "*distribution with regard to individual needs*"—, and formalize these as axioms on allocation rules: the "*Contribution Principle*" and the "*Needs Principle*" (Gotoh and Yoshihara (1997)). The *social decision procedure of fair allocation rules* (**SDPR**) is formalized as a function, which aggregates a profile of individual judgements over pairs of allocation rules and feasible allocations into a social judgement.[1] Through the **SDPR**, the society proceeds to choose an allocation rule, taking into account both *procedural* and *outcome-based* desirabilities of allocation rules.

In discussing **SDPR**, it is worth noting the importance of discriminating between individual *judgements* over possible rules and individual *tastes* over allocations. The domain of **SDPR** should be not a class of possible profiles of individual tastes, but a class of admissible profiles of individual judgements. Arrow (1963) himself also interpreted the domain of the Arrovian social welfare function as the class of profiles of individual judgements on what social situations are just or good. However, he defined individual judgements only as logically possible individual orderings over social situations, as he does for individual tastes. Following the original concern of Arrow (1963) on the process by which the society makes its choice, Pattanaik and Suzumura (1996) formalized the *extended social welfare function* as the social decision procedure to choose a right-structure of the society. They distinguished the domain over which individual judgements are defined from the domain of individual tastes. However, they still defined individual judgements only as logically possible individual orderings without any restriction. Thus, in their formulation, we cannot understand whether a revealed individual ordering may be represented as a self-interested taste on social outcomes or as a judgement based upon an opinion on what is social justice.

In this paper, as well as Arrow (1963) and Pattanaik and Suzumura

[1]This type of function is similar to an *extended social welfare function* (**ESWF**) (Pattanaik and Suzumura (1996)). However, as explained below, **SDPR** is a little different from **ESWF**.

(1996), we define an individual judgement as an ordering. In addition, however, we also propose some viewpoints from which an individual ordering should be classified into the class of judgements or of tastes. First, while a profile of individual tastes constitutes one aspect of an economic environment, a profile of individual judgements does not. It is because an individual judgement is to decide in what economic environments what allocation rules should be selected. In other words, an individual judgement states, for each economic environment, an ordering over possible allocation rules. Thus, an individual judgement should be defined mathematically as an *ordering function* of economic environments which include profiles of individual tastes. This implies that individual i's judgement can be interpreted as *i's proposing social welfare function,* which assigns to each economic environment an ordering over possible allocation rules. Second, it is conceptually reasonable to consider one's judgement as being revealed based upon her own opinion on what is social justice. This implies that individual judgements should be at least consistent with some "principle on justice or goodness." Thus, we define an individual judgement as the ordering function which satisfies at least some *axiom* that embodies an opinion on what is social justice. In a society which accepts the Rawlsian principles of justice, every individual judgement is regarded as satisfying at least several axioms which embody the Rawlsian principles. We define, in this paper, several axioms which seem to be consistent with the Rawlsian principles of justice, and impose them upon individual judgements over pairs of allocation rules and feasible allocations. Moreover, we say that a **SDPR** is Rawlsian one if its domain consists of the class of individual judgements which satisfy these axioms.

Before introducing these axioms, we should make mention of the Rawlsian difference principle. In discussing this principle, we should at least refer to the following problem: how to identify the least advantaged. This problem is related to which kind of index is appropriate for evaluating individual states. Notice that from the viewpoint of Rawls, we should not count individual preferences as such indices.[2] In this paper, we select *Sen's capability* index

[2] In past papers, the Rawlsian difference principle was formulated as a *traditional social welfare function* with *ordinal interpersonal comparison of individual welfare* (d'Aspremont and Gevers (1977), Hammond (1976, 1979), and Sen (1970, 1977) or as a *bargaining solution* (Binmore (1989) and Gauthier (1985)), although, as appropriate indices, Rawls himself used the *social primary goods* (Rawls (1971)) which are distinguished from individual welfare (preferences). Howe and Roemer (1981) and Roemer (1996) tried to formulate the difference principle faithful to the original discussion of Rawls.

(Sen (1980, 1985a,b)) as appropriate for evaluating individual states. Sen's capability is the set of an individual's various *relevant functioning* vectors which are possibly attainable by various utilizations of this person's share of resources.[3] Thus, in this paper, the least advantaged are identified as the persons who acquire the "minimum" expected capability in allocation.[4] Although determining what is the "minimum" expected capability is itself problematic, if there is at least the minimum in the sense of *set-inclusion* among capabilities of all individuals, we can state that this set is the "minimum" expected capability. Note that this minimal set is a *common capability* (Gotoh and Yoshihara (1997)) in the sense that it coincides with the intersection of all individuals' capabilities. Thus, in our model, the aim of the difference principle is reinterpreted as pursuing a guarantee to all individuals the "maximum" expected common capability.

As one of the Rawlsian axioms for individual judgements, we introduce *First Priority of Contribution Mechanisms* (**FPCM**), which relies on the *first principle of justice* of Rawls (1971) —*equal basic liberties.* The contribution mechanisms are allocation rules of a game form-type such that every individual's strategy is only in choosing her own labor time. This type of allocation rule guarantees every individual a right of freedom of choice.[5] **FPCM** requires individuals to judge that this type of allocation rule is morally better than any other type, irrelevant to what feasible allocations are realizable under those rules.

We also define *Second Priority of α-Combination Rules* (**SPCP**), *Priority of Feasible Pairs* (**PFP**), and *Priority of Equilibrium Outcomes* (**PEO**). **SPCP** privileges some types of allocation rules which *compromise* the two incompatible precepts, the Contribution and Needs Principles. **PFP** and **PEO** require individuals to judge that *realizable pairs* of rules and allocations are better than non-realizable ones. Particularly, **PEO** indicates that the social choice process of allocation rules should involve the society's prediction about what resource allocations will be realized.

As an axiom consistent with the difference principle, we propose the fifth

[3] Notice that this selection is permissible from the viewpoint of Rawls, because Rawls (1993) approved of using the capability index for evaluation of individual states.

[4] Roemer (1996) also proposed to adopt Sen's capability index in order to reformulate the difference principle.

[5] Gaertner, Pattanaik, and Suzumura (1992) discussed that the game form articulation of individual rights is more appropriate to represent individual rights of freedom of choice than the social choice correspondence articulation.

condition: *Consistency with Common Capability-Judgements* (**CCC**). This axiom requires every individual to judge that what should be socially chosen are realizable pairs, of which the allocation rules guarantee all individuals at least the "maximum" expected common capabilities. **CCC** also implies that every individual reveals, in her judgement ordering over possible realizable pairs, her own decision about what is the "maximum" expected common capability. To be consistent with the Rawlsian difference principle, a condition, *Set-Inclusion Subrelation*, should be imposed upon her judgement on this issue. This condition implies that any "maximum" expected common capability decided by any individual should be at least *maximal with respect to set-inclusion.*

The *Rawlsian social decision procedure of fair allocation rules* (**RSDPR**) is an **SDPR** which satisfies the *Pareto Principle*, **FPCM**, **SPCP**, **PFP**, **PEO**, and **CCC**. Through this function, once a profile of individual judgements on pairs of rules and feasible allocations is aggregated into a social judgement, the society will select, in each economic environment, an allocation rule. By **FPCM**, such a rule is a contribution mechanism. Note that by **SPCP**, this rule is a linear combination of two contribution mechanisms which meet the Contribution and Needs Principles respectively. Since this type of rule is a game form, a realizable pair in each economic environment, which is selected through the **RSDPR**, is the pair of this type of rule and some *ε-equilibrium allocation of the non-cooperative game* defined by this rule and the economic environment, whenever the equilibrium concept of this game is the ε-equilibrium one. This is followed by **PFP** and **PEO**. Moreover, through this non-cooperative game, the society can realize an allocation in which every individual is guaranteed at least the "maximal" common capability with respect to set-inclusion. Thus, once a social judgement is obtained through the **RSDPR**, the aim of the difference principle can be realized in each economic environment via such a decentralized manner.

The problem is then whether the **RSDPR** is well-defined or not. This solution depends upon the selection of two contribution mechanisms which meet the two precepts respectively, and the selection of the equilibrium concept of the non-cooperative games. In this paper, for the first selection problem, we adopt the *proportional sharing rule* (**PR**) and the **J**-*based capability maximin rule* ($\mathbf{CM}_J$) (Gotoh and Yoshihara (1997)). The former satisfies the Contribution Principle, while the latter, the Needs Principle. Thus, an allocation rule in an economy selected through the **RSDPR** is defined as a linear combination function of these two rules. For the second selection problem, we assume the *Nash equilibrium* concept. Thus, we discuss this problem by checking whether the non-cooperative games defined by the selected allocation rules have Nash equilibrium allocations or not.

There is still a serious problem with respect to the **RSDPR**. It is related to the existence of multiple equilibrium allocations of the non-cooperative game defined by the selected allocation rule. Under the allocation rule selected in an economic environment through the **RSDPR**, there may be two classes of non-cooperative equilibrium allocations. One consists of the allocations desirable from the viewpoint of the Rawlsian difference principle, while the other, if it is non-empty, consists of the non-desirable ones. Since both classes consist of equilibrium allocations, the society may realize the one in the non-desirable class as a result of the non-cooperative game. Then, the aim of the Rawlsian difference principle cannot be implemented, even if the selection of fair allocation rules in each economic environment is implemented through the **RSDPR**. We try to solve this realization problem by adopting the *mechanism design approach.*

In the remainder of this paper, section 2 defines a basic model. Section 3 defines the two precepts —the Contribution and Needs Principles—, and the **J**-based capability maximin rule. Section 4 introduces the **RSDPR**, and section 5 discusses whether or not the **RSDPR** is well-defined. Sections 6 discusses the above realization problem of the **RSDPR.**

2 The Basic Model

2.1 Economies and Feasible Allocations

The are two goods, one of which is labor time, $x \in \mathbb{R}_+$, utilized to produce the other, $y \in \mathbb{R}_+$. The population in a given society is $N = \{1, \cdots, n\}$ where $n \geqq 2$.[6] Individual i's consumption vector is denoted by $z_i = (l_i, y_i)$, where $l_i = \overline{x} - x_i$ denotes her leisure time and y_i denotes her share of output. In the society, all individuals have the same consumption set $[0, \overline{x}] \times \mathbb{R}_+$. Individual i is also characterized by utilization ability of resources, a_i, and some production skill, s_i. The universal set of utilization abilities for all

[6] We use vector inequalities: $\geqq$, $\geq$, $>$.

individuals is denoted by[7] $A \subseteq \mathbb{R}$. The universal set of production skills for all individuals is denoted by $S \subseteq \mathbb{R}_+$. Thus, individual i's objective characteristics are denoted by $(a_i, s_i) \in A \times S$.[8]

A production process is described by a production function $f : \mathbb{R}_+ \to \mathbb{R}_+$ which is assumed to be continuous, increasing, and $f(0) \geqq 0$. The set of such functions is denoted by F.

There are m types of *relevant functionings* for any individual, which are attainable by means of her leisure time and share of output. Let us assume that we can measure the achievements of these functionings by means of adequate indices. Thus, an achievement of functioning k is denoted by $b_k \in \mathbb{R}$. Individual i's achievement of relevant functionings is described by $b_i = (b_{i1}, \cdots, b_{im}) \in \mathbb{R}^m$.

Individual i's utilization ability, leisure time, and share of output influence the vector of functionings she can achieve. Let us assume that for each functioning k, there is a functional relationship between a triple of ability, leisure time, and share of output and its achievement: $c_k : A \times [0, \overline{x}] \times \mathbb{R}_+ \to \mathbb{R}$ such that $c_k(a, l, y) = b_k$. We call c_k functioning *k's utilization function.* For every functioning k, let c_k be concave and strictly monotonic on $A \times [0, \overline{x}] \times \mathbb{R}_+$, and given $a \in A$, $c_k(a, \cdot, \cdot)$ be continuous on $[0, \overline{x}] \times \mathbb{R}_+$. Moreover, for all $(a, l) \in A \times [0, \overline{x}]$, $c_k(a, l, 0) = 0$ and $\lim_{t \to \infty} \frac{c_k(a,l,ty)}{t} = 0$. Let $\mathcal{C}^m = \mathcal{C}_1 \times \cdots \times \mathcal{C}_m$ be the class of utilization functions satisfying the above properties. Given an ability $a \in A$, a profile of utilization functions $\mathbf{c} = (c_k)_{k \in \{1, \cdots, m\}} \in \mathcal{C}^m$, and resources $z = (l, y) \in [0, \overline{x}] \times \mathbb{R}_+$, *the capability under* (a, z), denoted by $C(a, z)$, is the set of functioning vectors achievable by various ways of utilizing z: $C(a, z) = \left\{ b \in \mathbb{R}^m \mid \exists\, \overline{z} = (\overline{z}^1, \cdots, \overline{z}^m), \sum_{k=1}^{m} \overline{z}^k \leqq z,\ c_k(a, \overline{z}^k) = b_k\ (\forall k) \right\}$.

By assumptions of c_k, the capability correspondence $C : A \times [0, \overline{x}] \times \mathbb{R}_+ \twoheadrightarrow \mathbb{R}^m$ has the following properties:

(1) For all $(a, l) \in A \times [0, \overline{x}]$, $C(a, l, 0) = \{0\}$.

(2) If $(a, z) \leqq (a', z')$, then $C(a, z) \subseteq C(a', z')$.

(3) For all $(a, z) \in A \times [0, \overline{x}] \times \mathbb{R}_+$, $C(a, z)$ is compact, comprehensive, and

[7] In the following, for any sets, X and X', 1) $X \supseteq X'$ if for all $x \in X'$, $x \in X$, 2) $X = X'$ if $X \supseteq X'$ and $X' \supseteq X$, 3) $not[X \supseteq X']$ if for some $x \in X'$, $x \notin X$, 4) $X \supsetneq X'$ if $X \supseteq X'$ but $not[X' \supseteq X]$, and 5) $not[X \supsetneq X']$ if $not[X \supseteq X']$ or $X' \supseteq X$.

[8] It may be natural to assume that there is a functional relationship between the production skill and the utilization ability such as $s_i = s(a_i)$. However, in this paper, we do not adopt such an assumption. All main results in this paper can be obtained regardless of whether or not that assumption is adopted.

convex in $\mathbb{R}^m$.
(4) given $a \in A$, $C : \{a\} \times [0, \overline{x}] \times \mathbb{R}_+ \twoheadrightarrow \mathbb{R}^m$ is continuous.

An objective characteristic of an economy is defined by a list $e = (\mathbf{a}, \mathbf{s}, f) = ((a_1, \cdots, a_n), (s_1, \cdots, s_n), f) \in E := A^n \times S^n \times F$. *A feasible allocation for* e is a vector $\mathbf{z} = (z_1, \cdots, z_n) \in ([0, \overline{x}] \times \mathbb{R}_+)^n$ such that $f(\sum_N s_i x_i) \geqq \sum_N y_i$. We denote by $Z(e)$ the set of feasible allocations for $e \in E$. Given an objective characteristic of an economy $e = (\mathbf{a}, \mathbf{s}, f) \in E$ and a profile of $\mathbf{c} = (c_k)_{k \in \{1, \cdots, m\}} \in \mathcal{C}^m$, *the feasible assignment of capabilities for* $(e, \mathbf{c})$ is a list $(C(a_1, z_1), \cdots, C(a_n, z_n))$ satisfying $(z_1, \cdots, z_n) \in Z(e)$.

For all $i \in N$, let $v_i : \mathbb{R}^m \to \mathbb{R}$ be a utility function satisfying continuous, strictly monotonic, and quasi-concave over $\mathbb{R}^m$. Let V be the class of utility functions satisfying the above three properties.

Given a profile of $\mathbf{c} = (c_k)_{k \in \{1, \cdots, m\}} \in \mathcal{C}^m$ and $z \in [0, \overline{x}] \times \mathbb{R}_+$, individual i endowed with $a_i \in A$ has an accessible set of functionings, $C(a_i, z)$. Then, the individual i endowed with $a_i \in A$ is *full-rational* if for all $z \in [0, \overline{x}] \times \mathbb{R}_+$, she always chooses $b_i^* \in C(a_i, z)$ such that for all $b_i \in C(a_i, z)$, $v_i(b_i^*) \geqq v_i(b_i)$[9]. In the following, we assume that every individual is full-rational. Given $e \in E$, $\mathbf{c} \in \mathcal{C}^m$, and $z \in [0, \overline{x}] \times \mathbb{R}_+$, let $b_i(e, \mathbf{c}, z, v_i) := \arg \max_{b_i \in C(a_i, z)} v_i(b_i)$.

2.2 Allocation Rules and Contribution Mechanisms as Game Forms

Let V^n be the n-fold Cartesian product of V. Let $Z(E) := \underset{e \in E}{\cup} Z(e)$. A *social choice correspondence* (SCC) is a *correspondence* $S : E \times \mathcal{C}^m \times V^n \twoheadrightarrow Z(E)$ such that for each environment $(e, \mathbf{c}, \boldsymbol{\nu}) \in E \times \mathcal{C}^m \times V^n$, $S(e, \mathbf{c}, \boldsymbol{\nu}) \subseteq Z(e)$. Let $\mathcal{S}$ be an admissible class of such correspondences.

A *mechanism* (or game form) is a pair $\Gamma = (M, g)$ where $M = M_1 \times \cdots \times M_n$, M_i is the s*trategy space of agent* i, and $g : E \times \mathcal{C}^m \times M \to ([0, \overline{x}] \times \mathbb{R}_+)^n$ is the *outcome function* which associate with each $(e, \mathbf{c}) \in E \times \mathcal{C}^m$ and $m \in M$ a unique element in $Z(e)$. Denote the i-th component of $g(e, \mathbf{c}, m)$ by $g_i(e, \mathbf{c}, m)$. Given $m \in M$ and $m_i' \in M_i$, (m_i', m_{-i}) is a strategy profile obtained by the replacement of m_i with m_i', and $g(e, \mathbf{c}, M_i, m_{-i})$ is the attainable set of feasible allocations that agent i can induce if other agents select m_{-i}. Let $M_{-i} := \times_{j \neq i} M_j$.

[9] In Sen′ s arguments on capability and well-being, individuals are not necessarily assumed to be full-rational. See Sen (1985a).

Given $e = (\mathbf{a}, \mathbf{s}, f) \in E$, a profile of $\mathbf{c} = (c_k)_{k\in\{1,\cdots,m\}} \in \mathcal{C}^m$, $\mathbf{v} = (v_i)_{i\in N} \in V^n$, and a mechanism $\Gamma = (M, g)$, a *non-cooperative game* is defined by $(N, e, \mathbf{c}, \mathbf{v}, \Gamma)$. Since in this paper, the set of players, N, is fixed, for simplicity, we denote one noncooperative game by $(e, \mathbf{c}, \mathbf{v}, \Gamma)$ only. Given $e = (\mathbf{a}, \mathbf{s}, f) \in E$, a profile of $\mathbf{c} = (c_k)_{k\in\{1,\cdots,m\}} \in \mathcal{C}^m$, a mechanism $\Gamma = (M, g)$, and a strategy profile $m \in M$, let $b_i(e, \mathbf{c}, m, \Gamma, v_i) := \arg \max_{b_i \in C(a_i, g_i(e,\mathbf{c},m))} v_i(b_i)$. Let $\mathbf{\Gamma}$ be an admissible class of such mechanisms.

Among various types of mechanisms, we are particularly concerned with the following type:

Definition 1: *The mechanism* $\Gamma = (M, g)$ *is a* contribution mechanism *if it satisfies the following properties:*

(1) for all $i \in N$, $M_i = [0, \overline{x}]$,

(2) for all $(e, \mathbf{c}) \in E \times \mathcal{C}^m$ *and all* $m = \mathbf{x} = (x_i)_{i\in N} \in M$, *for all* $i \in N$, $g_{i1}(e, \mathbf{c}, m) = \overline{x} - x_i$,

(3) for all $(e, \mathbf{c}) \in E \times \mathcal{C}^m$, $g(e, \mathbf{c}, M) \subseteq Z(e)$.

A contribution mechanism is one in which the strategy space of each agent is the set of labor time which is possibly chosen by herself, and the range of which is contained by the set of feasible allocations. This mechanism also meets the following property: in any economy, an agent i's supply of labor time is equivalent to her choice of strategy. This looks very desirable from the viewpoint of *equal basic liberties* (Rawls (1971)), since each agent can freely choose and realize her own labor time. Let $\mathbf{\Gamma}_{CO}$ be the subclass of $\mathbf{\Gamma}$, which consists of all possible contribution mechanisms. Note that any contribution mechanism is represented by a distribution rule (Gotoh and Yoshihara (1997)) defined as follows:

Definition 2 (Gotoh and Yoshihara (1997)): *A* distribution rule *is a function* $h : E \times \mathcal{C}^m \times [0, \overline{x}]^n \to \mathbb{R}^n_+$ *satisfying the following property: for any* $(e, \mathbf{c}) \in E \times \mathcal{C}^m$ *and any* $\mathbf{x} = (x_1, \cdots, x_n) \in [0, \overline{x}]^n$, $h(e, \mathbf{c}, x) = \mathbf{y} = (y_1, \cdots, y_n)$ *such that* $(\overline{x} - x_i, y_i)_{i\in N} \in Z(e)$.

Let $\mathcal{G} := \mathbf{\Gamma} \cup \mathcal{S}$ denote the class of all admissible *allocation rules* with generic element $\mathfrak{g}$. Note that when $\mathfrak{g} \in \mathcal{G}$, either $\mathfrak{g} \in \mathbf{\Gamma}$ or $\mathfrak{g} \in \mathcal{S}$.

3 Two Common Sense Precepts of Distributive Justice

In this section, we define two principles of distributive justice, the *Contribution* and *Needs Principles*, discussed by Gotoh and Yoshihara (1997). The two principles can be interpreted as expressions of *common sense precepts* (Rawls (1971)) of distributive justice. For simplicity, we define the above principles as ones which stipulate appropriate sets of *distribution rules* (Definition 2), as well as Gotoh and Yoshihara (1997).

First, we define the *Contribution Principle*:

Contribution Principle (Gotoh and Yoshihara (1997))[10] : *For all* $e = (\mathbf{a}, \mathbf{s}, f) \in E$, *all* $\mathbf{c} \in \mathcal{C}^m$, *and all* $\mathbf{x} = (x_1, \cdots, x_n) \in [0, \overline{x}]^n$, *the distribution of output assigned by the rule,* $h(e, \mathbf{x}) = \mathbf{y} = (y_1, \cdots, y_n)$ *satisfies:*

for all $i, j \in N$, $[s_i x_i < s_j x_j \Rightarrow h_i(e, \mathbf{c}, \mathbf{x}) < h_j(e, \mathbf{c}, \mathbf{x})$ & $s_i x_i = s_j x_j \Rightarrow h_i(e, \mathbf{c}, \mathbf{x}) = h_j(e, \mathbf{c}, \mathbf{x})]$.

Denote the class of distribution rules which satisfy Contribution Principle by $CR := \{h \in \mathbf{\Gamma}_{CO} \mid h$ satisfies *Contribution Principle*$\}$. There are many rules in CR. An example of one such rule is the *proportional sharing rule* PR, which distributes outputs in proportion to each one's labor contribution: for all $e = (\mathbf{a}, \mathbf{s}, f) \in E$, all $\mathbf{c} \in \mathcal{C}^m$, all $\mathbf{x} \in [0, \overline{x}]$, and all $i \in N$, $h_i^{PR}(e, \mathbf{c}, \mathbf{x}) = \frac{s_i x_i}{\sum s_j x_j} f(\sum s_j x_j)$.

The next principle we discuss is *Needs Principle,* which is defined as follows: given $e = (\mathbf{a}, \mathbf{s}, f) \in E$, $\mathbf{c} \in \mathcal{C}^m$, and $\mathbf{x} = (x_1, \cdots, x_n) \in [0, \overline{x}]^n$ such that for all $i, j \in N$, $x_i = x_j$, let $\mathbf{y}^H(e,\mathbf{x}) = (y_1, \cdots, y_n) = (\frac{f(\sum s_i x_i)}{n}, \cdots, \frac{f(\sum s_i x_i)}{n})$ be a *hypothetical distribution.* Then, a capability profile under $(e, \mathbf{x}, \mathbf{y}^H(e,\mathbf{x}))$ is determined as: $(C(a_1, \overline{x} - x_1, y^H(e,\mathbf{x})), \cdots, C(a_n, \overline{x} - x_n, y^H(e,\mathbf{x})))$. Moreover, by definition of the capability correspondence C, there is one individual $i^* \in N$ such that for any other $j \neq i^*$, $C(a_j, \overline{x} - x_j, y^H(e,\mathbf{x})) \supseteq C(a_{i^*}, \overline{x} - x_{i^*}, y^H(e,\mathbf{x}))$. Let $C^H(e,\mathbf{x}) := C(a_{i^*}, \overline{x} - x_{i^*}, y^H(e,\mathbf{x}))$, and call it a *reference capability under* $(e,\mathbf{x})$.

Needs Principle (Gotoh and Yoshihara (1997)) [11]: *For all* $e = (\mathbf{a}, \mathbf{s}, f) \in$

[10] Although this principle is defined as an axiom of distribution rules, it is easy to rewrite it so as to be applicable to other types of allocation rules.

[11] As well as the *Contribution Principle*, this principle is also rewritten to be applicable

E, *all* $\mathbf{c} \in \mathcal{C}^m$, *and all* $\mathbf{x} = (x_1, \cdots, x_n) \in [0, \overline{x}]^n$ *such that for all* $i, j \in N$, $x_i = x_j$, *the distribution of output assigned by the rule,* $h(e, \mathbf{c}, \mathbf{x}) = \mathbf{y} = (y_1, \cdots, y_n)$ *satisfies:*

there is no $i \in N$ *such that* $C^H(e,\mathbf{x}) \supsetneq C(a_i, \overline{x} - x_i, y_i)$.

Denote the class of distribution rules which satisfy Needs Principle by $NR := \{h \in \mathbf{\Gamma}_{CO} \mid h$ satisfies *Needs Principle*$\}$. The motivation of Needs Principle is briefly explained in Gotoh and Yoshihara (1997). Gotoh and Yoshihara (1997) also showed the general incompatibility of this principle with the Contribution Principle.

In this paper, we adopt the proportional sharing rule PR as the representative of Contribution Principle, while as a rule satisfying the Needs Principle, a **J**-*based capability maximin rule* which is introduced in Gotoh and Yoshihara (1997). In the following subsection, we define the class of **J**-*based capability maximin rules.*

3.1 The Class of J-Based Capability Maximin Rules

The class of **J**-*based capability maximin rules* (Gotoh and Yoshihara (1997)) is the class of distribution rules which distribute outputs so as to always guarantee every individual the "maximum" of *common capabilities.*

Given $e = (\mathbf{a}, \mathbf{s}, f)$ and $\mathbf{x} = (x_1, \cdots, x_n) \in [0, \overline{x}]^n$, let $Y(\mathbf{s}, f, \mathbf{x}) := \{\mathbf{y} = (y_1, \cdots, y_n) \in \mathbb{R}^n_+ \mid f(\sum_N s_i x_i) \geqq \sum_N y_i\}$ be a set of feasible distributions for $(e, \mathbf{x})$. Given $(e,\mathbf{c}) \in E \times \mathcal{C}^m$ and $\mathbf{x} = (x_1, \cdots, x_n) \in [0, \overline{x}]^n$, let $\mathcal{FC}(e,\mathbf{c}, \mathbf{x}) := \{(C(a_i, \overline{x} - x_i, y_i))_{i \in N} \mid \mathbf{y} = (y_i)_{i \in N} \in Y(\mathbf{s}, f, \mathbf{x})\}$ be a set of feasible assignments of capabilities under $(e,\mathbf{c}, \mathbf{x})$. For each $(C(a_i, \overline{x} - x_i, y_i))_{i \in N} \in \mathcal{FC}(e,\mathbf{c}, \mathbf{x})$, let $CC(e, \mathbf{c}, \mathbf{y}, \mathbf{x}) := \bigcap_{i \in N} C(a_i, \overline{x} - x_i, y_i)$ be a *common capability under* $(e,\mathbf{c}, \mathbf{x})$. Given $(e,\mathbf{c}) \in E \times \mathcal{C}^m$ and $\mathbf{x} = (x_1, \cdots, x_n) \in [0, \overline{x}]^n$, let $\mathcal{CC}(e,\mathbf{c}, \mathbf{x}) := \{CC(e,\mathbf{c}, \mathbf{y}, \mathbf{x}) \mid \mathbf{y} \in Y(\mathbf{s}, f, \mathbf{x})\}$ be the set of common capabilities under $(e,\mathbf{c}, \mathbf{x})$. Moreover, let $\mathcal{CC}(e,\mathbf{c}) := \bigcup_{\mathbf{x} \in [0,\overline{x}]^n} \mathcal{CC}(e,\mathbf{c}, \mathbf{x})$ and $\mathcal{CC} := \bigcup_{(e,\mathbf{c}) \in E \times \mathcal{C}^m} \mathcal{CC}(e,\mathbf{c})$.

In defining a **J**-*based capability maximin rule*, the determination of a "maximum" of *common capabilitiy* is arrived at by aggregating individual judgements on common capabilities to a social judgement. Suppose that such a judgement of individual i is represented by an ordering relation $J_i \subseteq$

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to not only the class of distribution rules, but also other classes of allocation rules.

$\mathcal{CC} \times \mathcal{CC}$. Let $J_i(e,\mathbf{c}) := J_i \cap [\mathcal{CC}(e,\mathbf{c}) \times \mathcal{CC}(e,\mathbf{c})]$ and $J_i(e,\mathbf{c},\mathbf{x}) := J_i(e,\mathbf{c}) \cap [\mathcal{CC}(e,\mathbf{c},\mathbf{x}) \times \mathcal{CC}(e,\mathbf{c},\mathbf{x})]$. For each $(e,\mathbf{c},\mathbf{x})$, one common capability in $\mathcal{CC}(e,\mathbf{c},\mathbf{x})$ is socially chosen by summarizing each profile of individuals' judgement $(J_i)_{i \in N}$ into a social ordering J. Moreover, in determing judgements, all individuals are assumed to have the following subrelation:[12]

Set-Inclusion Subrelations: *For all* $(e,\mathbf{c}) \in E \times \mathcal{C}^m$*, all* $i \in N$*, and all* $(\mathbf{x},\mathbf{y}),(\mathbf{x}^*,\mathbf{y}^*) \in Z(e)$*,*

$[CC(e,\mathbf{c},\mathbf{y},\mathbf{x}) \supseteq CC(e,\mathbf{c},\mathbf{y}^*,\mathbf{x}^*) \Rightarrow (CC(e,\mathbf{c},\mathbf{y},\mathbf{x}), CC(e,\mathbf{c},\mathbf{y}^*,\mathbf{x}^*)) \in J_i(e,\mathbf{c})]$.

Let $\mathcal{J}$ be the class of such orderings.

Let ψ be a social welfare function such that for every judgement profile $\mathbf{J} = (J_i)_{i \in N} \in \mathcal{J}^n$, $J = \psi(\mathbf{J})$ is an ordering. We assume that ψ meets the *Pareto Principle*: for all $\mathbf{J} = (J_i)_{i \in N}$, all $(e,\mathbf{x})$, and all $\mathbf{y},\mathbf{y}^* \in Y(\mathbf{s},f,\mathbf{x})$, if $(CC(e,\mathbf{c},\mathbf{y},\mathbf{x}), CC(e,\mathbf{c},\mathbf{y}^*,\mathbf{x})) \in J_i$ and $(CC(e,\mathbf{c},\mathbf{y}^*,\mathbf{x}), CC(e,\mathbf{c},\mathbf{y},\mathbf{x})) \notin J_i$ for all $i \in N$, then $(CC(e,\mathbf{c},\mathbf{y},\mathbf{x}), CC(e,\mathbf{c},\mathbf{y}^*,\mathbf{x})) \in \psi(\mathbf{J})$ and $(CC(e,\mathbf{c},\mathbf{y}^*,\mathbf{x}), CC(e,\mathbf{c},\mathbf{y},\mathbf{x})) \notin \psi(\mathbf{J})$. Thus, $\psi(\mathbf{J}) \in \mathcal{J}$ for every $\mathbf{J} \in \mathcal{J}^n$. Based upon a social welfare function $\psi(\mathbf{J})$, let us define a choice function φ as follows: for every $\mathbf{J} = (J_i)_{i \in N}$, $\varphi(\mathcal{CC}(e,\mathbf{c},\mathbf{x}),\psi(\mathbf{J})) \in \mathcal{CC}(e,\mathbf{c},\mathbf{x})$ and for all $\mathbf{y}' \in Y(\mathbf{s},f,\mathbf{x})$, $(\varphi(\mathcal{CC}(e,\mathbf{c},\mathbf{x}),\psi(\mathbf{J})), CC(e,\mathbf{c},\mathbf{y}',\mathbf{x})) \in \psi(\mathbf{J})$. Let $C^{\min}_{\varphi \cdot \psi(\mathbf{J})}(e,\mathbf{x}) := \varphi(\mathcal{CC}(e,\mathbf{c},\mathbf{x}),\psi(\mathbf{J}))$. We call this $C^{\min}_{\varphi \cdot \psi(\mathbf{J})}(e,\mathbf{x})$ a $\mathbf{J}$*-based minimal capability under* $(e,\mathbf{c},\mathbf{x})$.

Definition 3: *Given* ψ*,* φ*, and* $\mathbf{J} \in \mathcal{J}^n$*, the* $\mathbf{J}$-based capability maximin rule *(*$\mathbf{CM_J}$*) is a function* $h^{CM(\mathbf{J})} : E \times \mathcal{C}^m \times [0,\overline{x}]^n \to \mathbb{R}^n_+$ *such that for all* $e = (\mathbf{a},\mathbf{s},f) \in E$*, all* $\mathbf{c} \in \mathcal{C}^m$*, and all* $\mathbf{x} = (x_1,\cdots,x_n) \in [0,\overline{x}]^n$*,* $h^{CM(\mathbf{J})}(e,\mathbf{c},\mathbf{x}) = \mathbf{y} = (y_1,\cdots,y_n)$ *satisfies: for all* $i \in N$*,* $C(a_i,\overline{x}-x_i,y_i) \supseteq C^{\min}_{\varphi \cdot \psi(\mathbf{J})}(e,\mathbf{x})$.

Since under each $(e,\mathbf{c},\mathbf{x})$, $C^{\min}_{\varphi \cdot \psi(\mathbf{J})}(e,\mathbf{x})$ is diversified dependent upon the characteristic of $\mathbf{J} \in \mathcal{J}^n$, there are possibly multiple $\mathbf{J}$*-based capability maximin rules*, even if ψ and φ are fixed. Thus, we can define a class of possible $\mathbf{J}$*-based capability maximin rules*. Note that the class of all possible $\mathbf{J}$-based capability maximin distribution rules is invariantly independent of the char-

[12]It is worth noting that from the standpoint of Rawls, individual judgements about choosing the best common capability should be defined at least relatively independent of individual preferences $(\nu_i)_{i \in N}$ over functioning vectors. The following condition of *Set-Inclusion Subrelations* guarantees this relative independency.

acteristics of ψ and φ, as long as they satisfy the Pareto Principle. Hence, in what follows, we assume that ψ and φ are fixed, but not so for $\mathbf{J} \in \mathcal{J}^n$, and let $UCM := \{h^{CM(\mathbf{J})} \mid \mathbf{J} \in \mathcal{J}^n\}$ be the class of all possible **J**-based capability maximin rules. Gotoh and Yoshihara (1997) showed that any **J**-based capability maximin rule $(CM_{\mathbf{J}})$ is well-defined whenever ψ is continuous. They also showed that $UCM \subseteq NR$.

4 Social Choice of Allocation Rules according to the Rawlsian Two Principles of Justice

We regard the Rawlsian *two principles of justice* as the higher order principles which stipulate the social procedure of choosing *fair* allocation rules, while each of the other criteria on distributive justice, like the "no-envy" and "egalitarian-equivalent," discussed in the traditional social choice theory is regarded as only particular "*precepts*" in the framework of Rawls (1971). The Contribution and Needs Principles discussed in section 3 are two of such particular precepts. In this paper, we assume a society which accepts the Rawlsian principles of justice, and respects the Contribution and Needs Principles, both of which the difference principle compromises from the viewpoint of "the greatest expectation of the least advantaged."

The social procedure of choosing fair allocation rules is defined as a function which aggregates individual judgements on allocation rules into a social judgement.

4.1 Individual Judgements on Allocation Rules

Consider a situation in which among various allocation rules, a society selects some rules as fair ones, each of which is applied to some economic environment that will appear in the future of this society. In this situation, every individual cannot definitely know what types of economic environments will appear in the future, but she can know the class of possible economic environments. Then, taking into account which of the possible economic environments will appear, she is required to announce her own judgement on which of the allocation rules are fair. Consequently, a profile of all individuals' judgements is aggregated into a social judgement, according to which, allocation rules are chosen. After such a procedure of social choice of fair allocation rules, one economic environment prevails in this society, and one allocation rule is applied to it, according to the result of the social choice.

Thus, in the situation of socially choosing fair allocation rules, each individual reveals her judgement ordering, which is a function of possible economic environments. Note that in this paper, the class of possible economic environments is given as $E \times \mathcal{C}^m \times V^n$. As well as by regarding possible economic environments, some individual i may determine her own judgement on which of the allocation rules is fair, making it consistent with her judgement on what is the maximal common capability, $J_i \subseteq \mathcal{CC} \times \mathcal{CC}$. Consequently, it is natural to assume that any individual judgement on fair allocation rules is, in general, defined as an ordering function not only of possible economic environments, but also of admissible profiles of individual judgements on maximal common capabilities.

In what follows, a pair $(\mathbf{z}, \mathfrak{g})$ of $\mathbf{z} \in Z(E)$ and $\mathfrak{g} \in \mathcal{G}$ will be called an *extended social alternative.* Following the above discussion on individual judgements over allocation rules, we assume that, for all $i \in N$, individual i's judgement of elements of $Z(E) \times \mathcal{G}$ is given by an ordering correspondence $Q_i : E \times \mathcal{C}^m \times V^n \times \mathcal{J}^n \twoheadrightarrow (Z(E) \times \mathcal{G})^2$ which is defined as follows: for any $(e, \mathbf{c}, \mathbf{v}, \mathbf{J}) \in E \times \mathcal{C}^m \times V^n \times \mathcal{J}^n$, $Q_i(e, \mathbf{c}, \mathbf{v}, \mathbf{J}) \subseteq (Z(E) \times \mathcal{G})^2$ is an ordering. Denote the universal class of such judgement correspondences by $\mathcal{Q}$. We can interpret individual i's judgement correspondence, Q_i, as a *social welfare function,* which she would have proposed if she were the planner of the society. That is, we can consider a situation in which every individual proposes, based upon her own opinion about what is social justice, a social welfare function which always assigns to each economic environment and each judgement profile on common capabilities an ordering. Then, it is natural to introduce a social procedure in order to aggregate a profile $\mathbf{Q}(\cdot,\cdot,\cdot,\cdot) = (Q_i(\cdot,\cdot,\cdot,\cdot))_{i \in N} \in \mathcal{Q}^n$ into a social ordering $Q(\cdot,\cdot,\cdot,\cdot) \in \mathcal{Q}$.

Definition 4: *A* social decision procedure of fair allocation rules (**SDPR**) *is a function* Ψ *which maps each profile* $\mathbf{Q}(\cdot,\cdot,\cdot,\cdot) = (Q_i(\cdot,\cdot,\cdot,\cdot))_{i \in N}$ *of admissible judgement correspondences into one social judgement correspondence:* $Q(\cdot,\cdot,\cdot,\cdot) = \Psi(\mathbf{Q}(\cdot,\cdot,\cdot,\cdot))$.

Note that the domain of the SDPR, Ψ, the admissible class of judgement correspondences, does not necessarily coincide with $\mathcal{Q}$. What restrictions should be imposed in order to identify the admissible class of judgement correspondences depends upon what types of "distributive justice" each individual accepts as her own principles.

4.2 Definition of the Rawlsian Social Decision Procedure of Allocation Rules

In the Rawlsian framework, the social choice of allocation rules should be assumed to be processed under the veil of ignorance, subject to the imposition of the *two principles of justice.* The imposition of the two principles of justice implies, in our model, that the admissible set of each individual i's judgement $Q_i(\cdot, \cdot, \cdot, \cdot)$ does not possess the property of "*universal domain.*"

The first condition imposed upon $Q_i(\cdot, \cdot, \cdot, \cdot)$ is inspired by the *first principle of justice —equal basic liberties* (Rawls (1971)). In our setting, the first principle of justice by Rawls (1971) requires allocation rules to guarantee every individual the right of freedom to determine her own labor time. Since we assume that every individual accepts this principle, it is natural to impose upon each individual i's judgement, Q_i, the following condition:

First Priority of Contribution Mechanisms (FPCM): *For every individual $i \in N$, any $\mathfrak{g} \in \mathbf{\Gamma}_{CO}$, and any $\mathfrak{g}' \in \mathcal{G} \setminus \mathbf{\Gamma}_{CO}$,*

$[((\mathbf{z}, \mathfrak{g}), (\mathbf{z}', \mathfrak{g}')) \in Q_i(\cdot, \cdot, \cdot, \cdot) \& ((\mathbf{z}', \mathfrak{g}'), (\mathbf{z}, \mathfrak{g})) \notin Q_i(\cdot, \cdot, \cdot, \cdot)$ for all $\mathbf{z}$ and $\mathbf{z}'$ in $Z(E)]$.

Following **FPCM**, all individuals will reject all allocation rules in $\mathcal{G} \setminus \mathbf{\Gamma}_{CO}$, so that it will not be possible for these rules to be chosen by the society, whenever Ψ satisfies the Pareto Principle. However, many candidates still remain in $\mathbf{\Gamma}_{CO}$. The *second principle of justice* —the *difference principle* (Rawls (1971))— will work as a restriction.

Note that in this paper's model, there is no room for taking into account the first condition of the second principle of the Rawlsian justice —*fair equality of opportunity* (Rawls (1971))—, because every individual is endowed with fixed skill and ability and engages in the same type of labor.[13] Thus, after imposing the condition **FPCM** upon individual judgements, we can only concentrate on the *difference principle.* Herein, we consider the difference

[13] By fixing skills and abilities, we imply that there is no room for taking into account the opportunity of education, while by assuming only one type of labor, there is no room for taking into account the opportunity of occupation.

principle as the higher-order principle which appropriately compromises the two types of common sense precepts —the *Contribution* and *Needs Principles.* As one possible compromise-method of these precepts, we introduce a linear combination of the proportional and **J**-based capability maximin rules, although there may be other methods.

Given a judgement profile $\mathbf{J} = (J_i)_{i\in N} \in \mathcal{J}^n$ and $\alpha \in [0,1]$, let $h^{\alpha CP(\mathbf{J})}$ be an α-combination rule defined as follows: for all $e = (\mathbf{a}, \mathbf{s}, f) \in E$, all $\mathbf{c} \in \mathcal{C}^m$, all $\mathbf{x} \in [0, \overline{x}]$, and all $i \in N$, $h_i^{\alpha CP(\mathbf{J})}(e, \mathbf{c}, \mathbf{x}) = \alpha \cdot h_i^{CM(\mathbf{J})}(e, \mathbf{c}, \mathbf{x}) + (1 - \alpha) \cdot h_i^{PR}(e, \mathbf{c}, \mathbf{x})$. We introduce a new subclass of $\mathbf{\Gamma}_{CO}$ as follows: given $\mathbf{J} = (J_i)_{i\in N} \in \mathcal{J}^n$, let $CP(\mathbf{J}) := \{h^{\alpha CP(\mathbf{J})} \mid \alpha \in [0,1]\}$. Denote the universal class of α-combination rules by $CP := \underset{\mathbf{J}\in\mathcal{J}^n}{\cup} CP(\mathbf{J})$.

The second condition on $Q_i(\cdot,\cdot,\cdot,\cdot)$ is as follows:

Second Priority of α-Combination Rules (SPCP): *For every individual* $i \in N$, *any* $\mathfrak{g} \in CP$, *and any* $\mathfrak{g}' \in \mathbf{\Gamma}_{CO} \setminus CP$,

$[((\mathbf{z}, \mathfrak{g}), (\mathbf{z}', \mathfrak{g}')) \in Q_i(\cdot,\cdot,\cdot,\cdot) \& ((\mathbf{z}', \mathfrak{g}'), (\mathbf{z}, \mathfrak{g})) \notin Q_i(\cdot,\cdot,\cdot,\cdot)$ for all $\mathbf{z}$ and $\mathbf{z}'$ in $Z(E)]$.

Thus, the social choice problem of allocation rules has been reduced to the problem of choosing α-combination rules in order to be consistent with the requirement of the Rawlsian difference principle.

Next, we discuss the society's prediction about what resource allocations will be outcomes if one α-combination rule is granted. Given $(e, \mathbf{c}) \in E \times \mathcal{C}^m$, let denote the set of feasible pairs of allocations and α-combination rules under $(e, \mathbf{c})$ by $\mathcal{FP}(e, \mathbf{c}) := \{(\mathbf{z}, \mathfrak{g}) \in Z(E) \times \mathbf{\Gamma}_{CO} \mid \mathfrak{g}(e, \mathbf{c}, \mathbf{x}) = \mathbf{z}\}$. We then introduce the following condition:

Priority of Feasible Pairs (PFP): *For every individual* $i \in N$, *any* $\mathfrak{g}$ *and* $\mathfrak{g}' \in CP$, *any* $(e, \mathbf{c}) \in E \times \mathcal{C}^m$, *and* $\mathbf{z}$ *and* $\mathbf{z}'$ *in* $Z(E)$ *such that* $(\mathbf{z}, \mathfrak{g}) \in \mathcal{FP}(e, \mathbf{c})$ *and* $(\mathbf{z}', \mathfrak{g}') \notin \mathcal{FP}(e, \mathbf{c})$,

$[((\mathbf{z}, \mathfrak{g}), (\mathbf{z}', \mathfrak{g}')) \in Q_i(\cdot,\cdot,\cdot,\cdot) \& ((\mathbf{z}', \mathfrak{g}'), (\mathbf{z}, \mathfrak{g})) \notin Q_i(\cdot,\cdot,\cdot,\cdot)$ for all $\mathbf{z}$ and $\mathbf{z}'$ in $Z(E)]$.

Given $e = (\mathbf{a}, \mathbf{s}, f) \in E$, $\mathbf{c} \in \mathcal{C}^m$, $\mathfrak{g} \in \mathbf{\Gamma}$, and $\mathbf{v} \in V^n$, let $\varepsilon(e, \mathbf{c}, \mathbf{v}, \mathfrak{g})$ be the set of ε-equilibrium strategies of the non-cooperative game $(e, \mathbf{c}, \mathbf{v}, \mathfrak{g})$ and let the set of ε-equilibrium outcomes of the game be denoted by $T\varepsilon(e, \mathbf{c}, \mathbf{v}, \mathfrak{g})$. Define the following condition:

Priority of Equlibrium Outcomes (PEO): *For every individual* $i \in N$, *any* $(e, \mathbf{c}) \in E \times \mathcal{C}^m$, *any* $\mathbf{v} \in V^n$, *and any* $(\mathbf{z}, \mathfrak{g}), (\mathbf{z}', \mathfrak{g}') \in \mathcal{FP}(e, \mathbf{c})$ *such that* $\mathbf{z} \in T\varepsilon(e, \mathbf{c}, \mathbf{v}, \mathfrak{g})$ *and* $\mathbf{z}' \notin T\varepsilon(e, \mathbf{c}, \mathbf{v}, \mathfrak{g}')$,

$$[((\mathbf{z}, \mathfrak{g}), (\mathbf{z}', \mathfrak{g}')) \in Q_i(e, \mathbf{c}, \mathbf{v}, \cdot) \& ((\mathbf{z}', \mathfrak{g}'), (\mathbf{z}, \mathfrak{g})) \notin Q_i(e, \mathbf{c}, \mathbf{v}, \cdot)].$$

The next condition expresses a spirit of the difference principle:

Consistency with Common Capability-Judgements (CCC): *For every individual* $i \in N$, *any* $(e, \mathbf{c}) \in E \times \mathcal{C}^m$, *any* $\mathbf{v} \in V^n$, *any* $\mathbf{J} \in \mathcal{J}^n$, *and any* $(\mathbf{z}, \mathfrak{g}), (\mathbf{z}', \mathfrak{g}') \in \mathcal{FP}(e, \mathbf{c})$ *such that* $\mathbf{z} \in T\varepsilon(e, \mathbf{c}, \mathbf{v}, \mathfrak{g})$ *and* $\mathbf{z}' \in T\varepsilon(e, \mathbf{c}, \mathbf{v}, \mathfrak{g}')$,

$$[((\mathbf{z}, \mathfrak{g}), (\mathbf{z}', \mathfrak{g}')) \in Q_i(e, \mathbf{c}, \mathbf{v}, \mathbf{J}) \Leftrightarrow (CC(e, \mathbf{c}, \mathbf{z}), CC(e, \mathbf{c}, \mathbf{z}')) \in J_i].$$

That is, if individual i judges under the veil of ignorance, basing upon J_i, that the common capability in the extended alternative $(\mathbf{z}, \mathfrak{g})$ is better than that in $(\mathbf{z}', \mathfrak{g}')$, then she must judge that $(\mathbf{z}, \mathfrak{g})$ is better than $(\mathbf{z}', \mathfrak{g}')$ for any $(e, \mathbf{c}, \mathbf{v})$ in which both $(\mathbf{z}, \mathfrak{g})$ and $(\mathbf{z}', \mathfrak{g}')$ become equilibrium extended alternatives. Since by assumption, J_i includes set-inclusion as its subrelation, this condition implies that every individual should judge that what should be socially chosen is an extended alternative by which all individuals are guaranteed at least the common capability, which is "maximal" with respect to set-inclusion. Such a judgement is compatible with the Rawlsian difference principle. The reason is that the common capability is interpreted as the minimal capability which "the least advantaged" are at least guaranteed to acquire.

Definition 5: *Individual* i *accepts the* Rawlsian principles of justice *if her judgement corrspondence* $Q_i : E \times \mathcal{C}^m \times V^n \times \mathcal{J}^n \twoheadrightarrow (Z(E) \times \mathcal{G})^2$ *satisfies the conditions of* **FPCM**, **SPCP**, **PFP**, **PEO**, and **CCC**.

Denote the set of all possible judgement correspondences, $Q_i(\cdot, \cdot, \cdot, \cdot)$ of i who accepts the Rawlsian principle of justice by $\mathcal{Q}_R$, and the n-fold Cartesian product of $\mathcal{Q}_R$ by $\mathcal{Q}_R^n$. The Rawlsian social decision procedure of fair allocation rules is then defined as follows:

Definition 6: *The* Rawlsian social decision procedure of fair allocation rules (**RSDPR**) *is a function* $\Psi_R : \mathcal{Q}_R^n \to \mathcal{Q}_R$ *such that for every* $\mathbf{Q}(\cdot, \cdot, \cdot, \cdot) =$

$(Q_i(\cdot,\cdot,\cdot,\cdot))_{i\in N} \in \mathcal{Q}_R^n$, $\Psi_R(\mathbf{Q}(\cdot,\cdot,\cdot,\cdot)) = Q(\cdot,\cdot,\cdot,\cdot)$, *and* Ψ_R *satisfies the* Pareto Principle.

Given $(e, \mathbf{c}, \mathbf{v}, \mathbf{J}) \in E \times \mathcal{C}^m \times V^n \times \mathcal{J}^n$, let us denote the set of socially best extended alternatives determined through Ψ_R under $\mathbf{Q}(\cdot,\cdot,\cdot,\cdot) \in \mathcal{Q}_R^n$ by $B_{\Psi_R(\mathbf{Q})}(e, \mathbf{c}, \mathbf{v}, \mathbf{J}) := \{(\mathbf{z}, \mathfrak{g}) \in Z(E) \times \mathcal{G} \mid ((\mathbf{z}, \mathfrak{g}), (\mathbf{z}', \mathfrak{g}')) \in \Psi_R(\mathbf{Q}(e, \mathbf{c}, \mathbf{v}, \mathbf{J}))$ for all $(\mathbf{z}', \mathfrak{g}') \in Z(E) \times \mathcal{G}\}$. Thus, given Ψ_R and a profile of individual judgements $\mathbf{Q}(\cdot,\cdot,\cdot,\cdot) \in \mathcal{Q}_R^n$, if $(e, \mathbf{c}) \in E \times \mathcal{C}^m$ is the current objective environment, and $\mathbf{v} \in V^n$ and $\mathbf{J} \in \mathcal{J}^n$ are revealed by individuals, then the set of allocation rules socially chosen by Ψ_R is: $D_{\Psi_R(\mathbf{Q})}(e, \mathbf{c}, \mathbf{v}, \mathbf{J}) := \{\mathfrak{g} \in \mathcal{G} \mid \exists \mathbf{z} \in Z(E),\ s.t.\ (\mathbf{z}, \mathfrak{g}) \in B_{\Psi_R(\mathbf{Q})}(e, \mathbf{c}, \mathbf{v}, \mathbf{J})\}$. This implies that the allocation rule $\mathfrak{g}$ is possibly selected through $\Psi_R(\mathbf{Q}(\cdot,\cdot,\cdot,\cdot))$ in $(e, \mathbf{c}, \mathbf{v}, \mathbf{J})$.

A specific characteristic of Ψ_R is that it requires all individuals to restrict their admissible sets of individual judgements over extended alternatives to the ones satisfying the above five conditions. It may be a curious approach from the viewpoint of canonical social choice theories in which the universal domain is usually assumed. However, from the viewpoint of the Rawlsian two principles of justice, it is not odd, since we assume a society in which every individual agrees to accept the Rawlsian two principles of justice. Notice that the problems of why and how the society agrees to accept these principles is itself another issue separated from this paper's theme.

Some may question whether these five conditions are effective to restrict the sets of individual judgements, even if such a restriction is justified from the viewpoint of Rawls. It is because individual judgements are private information, so that each individual can arbitrarily reveal her own judgement without any punishment. However, the answer to this question is Yes ! The above five conditions are effective in the sense that no individual can arbitrarily reveal her own judgement without any punishment. The reason is that whether or not any ordered pair of any two extended alternatives meets the four conditions except **CCC** can be checked objectively. With respect to **CCC**, also, if $CC(e, \mathbf{c}, \mathbf{z}) \subsetneq CC(e, \mathbf{c}, \mathbf{z}')$ for $(e, \mathbf{c}) \in E \times \mathcal{C}^m$ and $\mathbf{z}, \mathbf{z}' \in Z(e)$, but there is someone i such that for some $\mathbf{v} \in V^n$ and $\mathbf{J} \in \mathcal{J}^n$, $\mathbf{z} \in T\varepsilon(e, \mathbf{c}, \mathbf{v}, \mathfrak{g})$, $\mathbf{z}' \in T\varepsilon(e, \mathbf{c}, \mathbf{v}, \mathfrak{g}')$, and $((\mathbf{z}', \mathfrak{g}'), (\mathbf{z}, \mathfrak{g})) \notin Q_i(e, \mathbf{c}, \mathbf{v}, \mathbf{J})$, then i will be surely punished, because the set-inclusion $CC(e, \mathbf{c}, \mathbf{z}) \subsetneq CC(e, \mathbf{c}, \mathbf{z}')$ is observed.

4.3 Characterizing the Set of Best Extended Alternatives determined by Ψ_R

Given Ψ_R, $\mathbf{Q}(\cdot,\cdot,\cdot,\cdot) \in \mathcal{Q}_R^n$, $\mathbf{J} \in \mathcal{J}^n$, $(e,\mathbf{c}) \in E \times \mathcal{C}^m$, and $\mathbf{v} \in V^n$, let $(\mathbf{z},\mathfrak{g}) \in B_{\Psi_R(\mathbf{Q})}(e,\mathbf{c},\mathbf{v},\mathbf{J})$. By **FPCM** and **SPCP**, this allocation rule $\mathfrak{g}$ is an α-combination rule: there is $h^{\alpha CP(\mathbf{J})} \in CP(\mathbf{J})$ such that $h^{\alpha CP(\mathbf{J})} = \mathfrak{g}$. By **PFP** and **PEO**, $\mathbf{z} \in T\varepsilon(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})$.

Note that for each $\alpha' \in [0,1]$ and $\mathbf{z}' \in T\varepsilon(e,\mathbf{c},\mathbf{v},h^{\alpha' CP(\mathbf{J})})$, $CC(e,\mathbf{c},\mathbf{z}')$ is uniquely determined. Let $\mathcal{CC}(T\varepsilon(e,\mathbf{c},\mathbf{v},h^{\alpha' CP(\mathbf{J})})) := \{CC(e,\mathbf{c},\mathbf{z}') \mid \mathbf{z}' \in T\varepsilon(e,\mathbf{c},\mathbf{v},h^{\alpha' CP(\mathbf{J})})\}$. Then, given $\alpha' \in [0,1]$, $e = (\mathbf{a},\mathbf{s},f) \in E$, $\mathbf{c} \in \mathcal{C}^m$, and $\mathbf{v} \in V^n$, let $\mathcal{Z}\varepsilon(e,\mathbf{c},\mathbf{v},h^{\alpha' CP(\mathbf{J})}) := \{\mathbf{z}' \in T\varepsilon(e,\mathbf{c},\mathbf{v},h^{\alpha' CP(\mathbf{J})}) \mid \forall \mathbf{z}^* \in T\varepsilon(e,\mathbf{c},\mathbf{v},h^{\alpha' CP(\mathbf{J})}), (CC(e,\mathbf{c},\mathbf{z}'),CC(e,\mathbf{c},\mathbf{z}^*)) \in \psi(\mathbf{J})\}$. Notice that if both $\mathbf{z}$ and $\mathbf{z}'$ belong to $\mathcal{Z}\varepsilon(e,\mathbf{c},\mathbf{v},h^{\alpha' CP(\mathbf{J})})$, then $CC(e,\mathbf{c},\mathbf{z})$ and $CC(e,\mathbf{c},\mathbf{z}')$ are indifferent with respect to $\psi(\mathbf{J})$. Let $\{\mathcal{Z}\varepsilon(e,\mathbf{c},\mathbf{v},h^{\alpha' CP(\mathbf{J})})\}_{\alpha' \in [0,1]}$. Note that if for some α' and α^* in $[0,1]$, there exist $\mathbf{z}' \in \mathcal{Z}\varepsilon(e,\mathbf{c},\mathbf{v},h^{\alpha' CP(\mathbf{J})})$ and $\mathbf{z}^* \in \mathcal{Z}\varepsilon(e,\mathbf{c},\mathbf{v},h^{\alpha^* CP(\mathbf{J})})$ such that $(CC(e,\mathbf{c},\mathbf{z}'),CC(e,\mathbf{c},\mathbf{z}^*)) \in \psi(\mathbf{J})$, then for all $\mathbf{z}'' \in \mathcal{Z}\varepsilon(e,\mathbf{c},\mathbf{v},h^{\alpha' CP(\mathbf{J})})$ and all $\mathbf{z}^{*\prime} \in \mathcal{Z}\varepsilon(e,\mathbf{c},\mathbf{v},h^{\alpha^* CP(\mathbf{J})})$, $(CC(e,\mathbf{c},\mathbf{z}''),CC(e,\mathbf{c},\mathbf{z}^{*\prime})) \in \psi(\mathbf{J})$. For each $\alpha' \in [0,1]$, denote one element in $\mathcal{Z}\varepsilon(e,\mathbf{c},\mathbf{v},h^{\alpha' CP(\mathbf{J})})$ by $\mathbf{z}\varepsilon(e,\mathbf{c},\mathbf{v},h^{\alpha' CP(\mathbf{J})})$. Given $e = (\mathbf{a},\mathbf{s},f) \in E$, $\mathbf{c} \in \mathcal{C}^m$, and $\mathbf{v} \in V^n$, let $\mathcal{A}\varepsilon(e,\mathbf{c},\mathbf{v},CP(\mathbf{J})) := \{\alpha' \in [0,1] \mid \forall \alpha^* \in [0,1], (CC(e,\mathbf{c},\mathbf{z}\varepsilon(e,\mathbf{c},\mathbf{v},h^{\alpha' CP(\mathbf{J})})),CC(e,\mathbf{c},\mathbf{z}\varepsilon(e,\mathbf{c},\mathbf{v},h^{\alpha^* CP(\mathbf{J})}))) \in \psi(\mathbf{J})\}$. Let $\mathcal{A}\varepsilon(CP(\mathbf{J})) := \bigcup_{(e,\mathbf{c},\mathbf{v}) \in E \times \mathcal{C}^m \times V^n} \mathcal{A}\varepsilon(e,\mathbf{c},\mathbf{v},CP(\mathbf{J}))$.

By **CCC**, $(\mathbf{z},h^{\alpha CP(\mathbf{J})}) \in B_{\Psi_R(\mathbf{Q})}(e,\mathbf{c},\mathbf{v},\mathbf{J})$ implies that $\mathbf{z} \in \mathcal{Z}\varepsilon(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})$ and $\alpha \in \mathcal{A}\varepsilon(e,\mathbf{c},\mathbf{v},CP(\mathbf{J}))$. Thus, if individuals' judgements on extended alternatives are $\mathbf{Q}(\cdot,\cdot,\cdot,\cdot) \in \mathcal{Q}_R^n$, and their judgements on common capabilities are $\mathbf{J} \in \mathcal{J}^n$, then, in each economic environment $(e,\mathbf{c},\mathbf{v}) \in E \times \mathcal{C}^m \times V^n$, some α-combination rule $h^{\alpha CP(\mathbf{J})}$ such that $\alpha \in \mathcal{A}\varepsilon(e,\mathbf{c},\mathbf{v},CP(\mathbf{J}))$ will be selected by $\Psi_R(\mathbf{Q}(\cdot,\cdot,\cdot,\cdot))$. Once the information of an economic environment $(e,\mathbf{c},\mathbf{v}) \in E \times \mathcal{C}^m \times V^n$ prevails in the society, and an α-combination rule $h^{\alpha CP(\mathbf{J})}$ such that $\alpha \in \mathcal{A}\varepsilon(e,\mathbf{c},\mathbf{v},CP(\mathbf{J}))$ is selected in $(e,\mathbf{c},\mathbf{v})$ through $\Psi_R(\mathbf{Q}(\cdot,\cdot,\cdot,\cdot))$, then some feasible allocation $\mathbf{z} \in \mathcal{Z}\varepsilon(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})$ will be realizable as an equilibrium outcome of the non-cooperative game $(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})$. Note that the set $B_{\Psi_R(\mathbf{Q})}(e,\mathbf{c},\mathbf{v},\mathbf{J})$ is not necessarily singleton. By **CCC**, however, for all $(\mathbf{z},h^{\alpha CP(\mathbf{J})})$ and $(\mathbf{z}',h^{\alpha' CP(\mathbf{J})})$ in $B_{\Psi_R(\mathbf{Q})}(e,\mathbf{c},\mathbf{v},\mathbf{J})$, $(CC(e,\mathbf{c},\mathbf{z}),CC(e,\mathbf{c},\mathbf{z}')) \in \psi(\mathbf{J})$ and $(CC(e,\mathbf{c},\mathbf{z}'),CC(e,\mathbf{c},\mathbf{z})) \in \psi(\mathbf{J})$. This implies that any $(\mathbf{z},h^{\alpha CP(\mathbf{J})})$ and $(\mathbf{z}',h^{\alpha' CP(\mathbf{J})})$ in $B_{\Psi_R(\mathbf{Q})}(e,\mathbf{c},\mathbf{v},\mathbf{J})$ are indifferent from the viewpoint of the Rawlsian difference principle.

By using the *Rawlsian social decision correspondence of fair allocation* *rules,* $D_{\Psi_R(\cdot)}$, we can also define a new allocation rule as follows:

Definition 7: *Given* ψ, φ, *and* $\mathbf{J} \in \mathcal{J}^n$, *the* **J**-based difference solution *is a* correspondence $S^{\mathcal{A}\varepsilon(CP(\mathbf{J}))} : E \times \mathcal{C}^m \times V^n \twoheadrightarrow Z(E)$ *such that for all* $(e,\mathbf{c},\mathbf{v}) \in E \times \mathcal{C}^m \times V^n$, $S^{\mathcal{A}\varepsilon(CP(\mathbf{J}))}(e,\mathbf{c},\mathbf{v}) = \bigcup_{\alpha \in \mathcal{A}\varepsilon(e,\mathbf{c},\mathbf{v},CP(\mathbf{J}))} \mathcal{Z}\varepsilon(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})$.

The **J**-based difference solution represents an aim of the *Rawlsian difference principle* (Rawls (1971)). However, notice that the aim of the *Rawlsian difference principle* is realized not by the class of **J**-based difference solutions, but by $D_{\Psi_R(\cdot)}$.

5 Well-definedness of the Rawlsian Social Decision Procedure of Fair Allocation Rules

We discuss, in this section, an issue of well-definedness of the **RSDPR**. If for all $(e,\mathbf{c},\mathbf{v},\mathbf{J})$ and all $\mathbf{Q}(\cdot,\cdot,\cdot,\cdot)$, $D_{\Psi_R(\cdot)}$ is empty, then it is natural to say that the **RSDPR** is not well-defined. This problem can be reduced to that of non-emptiness of **J**-based difference solutions. The solution of this non-emptiness problem depends upon the selection of equilibrium concepts in non-cooperative games. In the following, we check the non-emptiness of **J**-based difference solutions by adopting a pure-strategy Nash equilibrium concept. Given $\alpha \in [0,1]$, $e = (\mathbf{a},\mathbf{s},f) \in E$, $\mathbf{c} \in \mathcal{C}^m$, and $\mathbf{v} \in V^n$, let $NE(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})$ and $NA(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})$ be the sets of pure-strategy Nash equilibria and Nash equilibirum allocations of the non-cooperative game $(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})$.

For a given $e = (\mathbf{a},\mathbf{s},f) \in E$, $\mathbf{c} \in \mathcal{C}^m$, and $\mathbf{v} \in V^n$, we must show that there is at least one $\alpha \in [0,1]$ such that $NA(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})$ is non-empty and compact. If it does so, then $S^{\mathcal{A}(CP)}(e,\mathbf{c},\mathbf{v})$ is non-empty if $\psi(\mathbf{J})$ is continuous ordering.

Given $(e,\mathbf{c},h)$, let $\mathbf{C}^h_e : [0,\overline{x}]^n \twoheadrightarrow \mathbb{R}^{mn}$ be such that $\mathbf{C}^h_e(\mathbf{x}) = \{(b_i)_{i\in N} \in \mathbb{R}^{mn} \mid \forall i \in N,\ b_i \in C(a_i,\overline{x}-x_i,h_i(e,\mathbf{c},\mathbf{x}))\}$. Given a non-cooperative game $(e,\mathbf{c},\mathbf{v},h)$, we can define a payoff function for each $i \in N$, $u^h_i : [0,\overline{x}]^n \times \mathbf{C}^h_e([0,\overline{x}]^n) \to \mathbb{R}$ such that $u^h_i(\mathbf{x},\mathbf{b}) = v_i(b_i)$ where $b_i \in C(a_i,\overline{x}-x_i,h_i(e,\mathbf{c},\mathbf{x}))$. Notice that if every individual is full-rational, she chooses $b_i = b_i(e,\mathbf{x},h,v_i)$. Let F^c be a subset of F such that for any $f \in F^c$, f is concave. Then, let $E^c := A^n \times S^n \times F^c$. The following proposition shows that if $e \in E^c$, then for any $\mathbf{c} \in \mathcal{C}^m$ and $\mathbf{v} \in V^n$, $NA(e, \mathbf{c}, \mathbf{v}, h^{PR})$ is non-empty.

Proposition 1: *For all $\mathbf{c} \in \mathcal{C}^m$, all $\mathbf{v} \in V^n$, and all $e \in E^c$, $NA(e, \mathbf{c}, \mathbf{v}, h^{PR}) \neq \varnothing$.*

Proof: See Appendix.

We can also prove that under some restricted economic environments, the non-cooperative game defined by **J**-based capability maximin rule has a non-empty set of pure-strategy Nash equilibrium allocations. Given ψ, $\mathbf{J} \in \mathcal{J}^n$, $e \in E$, $\mathbf{c} \in \mathcal{C}^m$, and $\mathbf{x} \in [0, \overline{x}]^n$, let $\mathcal{B}(\mathcal{CC}(e, \mathbf{c}, \mathbf{x}), \psi(\mathbf{J})) := \{CC(e, \mathbf{c}, \mathbf{y}^*, \mathbf{x}) \in \mathcal{CC}(e, \mathbf{c}, \mathbf{x}) \mid \forall CC(e, \mathbf{c}, \mathbf{y}, \mathbf{x}) \in \mathcal{CC}(e, \mathbf{c}, \mathbf{x}), (CC(e, \mathbf{c}, \mathbf{y}^*, \mathbf{x}), CC(e, \mathbf{c}, \mathbf{y}, \mathbf{x})) \in \psi(\mathbf{J})\}$. Note that $\varphi(\mathcal{CC}(e, \mathbf{c}, \mathbf{x}), \psi(\mathbf{J})) \in \mathcal{B}(\mathcal{CC}(e, \mathbf{c}, \mathbf{x}), \psi(\mathbf{J}))$. In the rest of this paper, we rely on the following assumption:

Assumption 1: *For all $\mathbf{J} \in \mathcal{J}^n$, all $e \in E$, and all $\mathbf{x} \in [0, \overline{x}]^n$, $\mathcal{B}(\mathcal{CC}(e, \mathbf{c}, \mathbf{x}), \psi(\mathbf{J}))$ is singleton.*

Assumption 2: For all $a \in A$ and all $z, z' \in [0, \overline{x}] \times \mathbb{R}_+$, if for some functioning $k \in \{1, \cdots, m\}$, $c_k(a, z) \leq c_k(a, z')$, then for any other functioning $k' \in \{1, \cdots, m\}$, $c_{k'}(a, z) \leq c_{k'}(a, z')$.

By Assumption 2, we can induce that if for some functioning $k \in \{1, \cdots, m\}$, $c_k(a, z) =$ (resp. $<$) $c_k(a, z')$, then for any other functioning $k' \in \{1, \cdots, m\}$, $c_{k'}(a, z) =$ (resp.$<$) $c_{k'}(a, z')$. This assumption states that all functionings have the same "*isoquant curves*" of their utilization functions. Notice that this does not imply that all functionings' utilization functions have the same shape, because some functioning k's utilization function c_k may be homogeneous of degree one, while other's $c_{k'}$ may be strictly concave.[14]

Assumption 3: For all $k \in \{1, \cdots, m\}$, all $a \in A$, and all $z, z' \in [0, \overline{x}] \times \mathbb{R}_+$, if $c_k(a, z) = c_k(a, z')$, then for any $\lambda \in \mathbb{R}_+$, $c_k(a, \lambda z) = c_k(a, \lambda z')$.

[14]Note that every functioning is cardinally measurable. Hence, even if two functionings have the same "isoquant curves" of their utilization functions, the difference in the shape of utilization functions between these two reflect their difference with respect to the characteristics of these two functionings.

This assumption is also pertinent to utilization functions of relevant functionings. This says that all possible utilization functions are homothetic.

Assumption 4: For all $\mathbf{J} \in \mathcal{J}^n$, all $e \in E$, and all $\mathbf{x}, \mathbf{x}' \in [0, \overline{x}]^n$ such that $\mathbf{x} \leq \mathbf{x}'$, $\varphi(\mathcal{CC}(e, \mathbf{x}), \psi(\mathbf{J}))$ and $\varphi(\mathcal{CC}(e, \mathbf{x}'), \psi(\mathbf{J}))$ satisfy the following property: if for some $i \in N$, $C(a_i, \overline{x} - x_i, h_i^{CM(\mathbf{J})}(e, \mathbf{x})) \subsetneq$ (resp. $\supsetneq$) $C(a_i, \overline{x} - x_i', h_i^{CM(\mathbf{J})}(e, \mathbf{x}'))$, then *not* $[C(a_j, \overline{x} - x_j, h_j^{CM(\mathbf{J})}(e, \mathbf{x})) \supsetneq$ (resp.$\subsetneq$) $C(a_j, \overline{x} - x_j', h_j^{CM(\mathbf{J})}(e, \mathbf{x}'))$ for some $j \neq i]$.

This assumption is a condition imposed upon the capability maximin rule. It is a solidarity condition. It says that changing production activity should not be effective in opposite directions between some individuals.

Proposition 2: *Under Assumption 1, 2, 3, and 4, if the social welfare function* ψ *assigns continuous ordering on* $\mathcal{CC}$*, then for all* $\mathbf{J} \in \mathcal{J}^n$*, all* $e = (\mathbf{a}, \mathbf{s}, f) \in E^C$*, a pure-strategy Nash equilibrium of the non-cooperative game* $(\mathbf{v}, h^{CM(\mathbf{J})})$ *under* $e \in E^C$ *exists.*

Proof. See Gotoh and Yoshihara (1997).

Let $NA^{-1}(h^{CM(\mathbf{J})}) := \{(e, \mathbf{c}, \mathbf{v}) \in E \times \mathcal{C}^m \times V^n \mid NA(e, \mathbf{c}, \mathbf{v}, h^{CM(\mathbf{J})}) \neq \varnothing\}$ and $NA^{-1}(h^{PR}) := \{(e, \mathbf{c}, \mathbf{v}) \in E \times \mathcal{C}^m \times V^n \mid NA(e, \mathbf{c}, \mathbf{v}, h^{PR}) \neq \varnothing\}$. By Propositions 1 and 2 of this paper, both $NA^{-1}(h^{CM(\mathbf{J})})$ and $NA^{-1}(h^{PR})$ are non-empty. Let $NA_c^{-1}(h^{CM(\mathbf{J})}) := \{(e, \mathbf{c}, \mathbf{v}) \in NA^{-1}(h^{CM(\mathbf{J})}) \mid e \in E^c\}$ and $NA_c^{-1}(h^{PR}) := \{(e, \mathbf{c}, \mathbf{v}) \in NA^{-1}(h^{PR}) \mid e \in E^c\}$. By Proposition 1 of this paper, $NA_c^{-1}(h^{PR}) = E^c \times \mathcal{C}^m \times V^n$. Since by Proposition 2, $NA_c^{-1}(h^{CM(\mathbf{J})}) \neq \varnothing$, we obtain that $NA^{-1}(h^{CM(\mathbf{J})}) \cap NA^{-1}(h^{PR}) \neq \varnothing$.

Theorem 1: *Assume the pure-strategy Nash equilibrium concept. Then, under Assumption 1, if* $\psi(\mathbf{J})$ *is continuous, then for all* $(e, \mathbf{c}, \mathbf{v}) \in NA^{-1}(h^{CM(\mathbf{J})}) \cup NA^{-1}(h^{PR})$, $S^{A(CP(\mathbf{J}))}(e, \mathbf{c}, \mathbf{v})$ *is non-empty.*

Proof. See Appendix.

Corollary 1: *Assume the pure-strategy Nash equilibrium concept. Then, under Assumption 1, if* $\psi(\mathbf{J})$ *is continuous, then for all* $(e, \mathbf{c}, \mathbf{v}) \in NA^{-1}(h^{CM(\mathbf{J})}) \cup$

$NA^{-1}(h^{PR})$, $D_{\Psi_R(\mathbf{Q})}(e, \mathbf{c}, \mathbf{v}, \mathbf{J})$ *is non-empty for any* $\mathbf{Q} \in \mathcal{Q}_R^n$.

Proof. See Appendix.

By Corollary 1, the **RSDPR** is well-defined in a rather broader class of economic environments if we assume the pure-strategy Nash equilibrium behavior. Note that for some economic environment and some judgement profile, the selected **J**-based α-combination rule may be only the proportional rule (that is, $\alpha = 0$), because there may be no other **J**-based α-combination rule which has a Nash equilibrium allocation in this economy. We cannot exclude such an extreme case if we apply the **RSDPR** to the broader class of economies. However, in a subclass of economies, $E^c \times \mathcal{C}^m \times V^n$, which meets the Assumptions 2, 3, and 4, any **J**-based α -combination rule has at least one Nash equilibrium. This is because under the Assumptions 2, 3, and 4, both of $C(a_i, \cdot, h_i^{PR}(\cdot, x_{-i}))$ and $C(a_i, \cdot, h_i^{CM(\mathbf{J})}(\cdot, x_{-i}))$ have a convex graph in $E^c \times \mathcal{C}^m \times V^n$. Thus, in this case, the weight value, α, of the selected **J**-based α-combination rule may be more than zero.

6 The "Moral Hazard" Problem in the Rawlsian Social Decision Procedure of Allocation Rules

As discussed in the previous sections, if individuals' judgements on extended alternatives are $\mathbf{Q}(\cdot,\cdot,\cdot,\cdot) \in \mathcal{Q}_R^n$, and their judgements on common capabilities are $\mathbf{J} \in \mathcal{J}^n$, then, in each economic environment $(e, \mathbf{c}, \mathbf{v}) \in E \times \mathcal{C}^m \times V^n$, some **J**-based α-combination rule $h^{\alpha CP(\mathbf{J})}$ such that $\alpha \in \mathcal{A}\varepsilon(e, \mathbf{c}, \mathbf{v}, CP(\mathbf{J}))$ will be selected by $\Psi_R(\mathbf{Q}(\cdot,\cdot,\cdot,\cdot))$. Thus, the society can select an allocation rule which can realize the aim of the Rawlsian difference principle: to guarantee every individual at least a common capability which is maximal with respect to set-inclusion.

However, there still remains a problem. Even if the true information of $(e, \mathbf{c}, \mathbf{v})$ is revealed in the society, and an appropriate α-combination rule $h^{\alpha CP(\mathbf{J})}$ is selected, individuals may not realize the feasible allocations which are desirable from the viewpoint of the Rawlsian difference principle. This problem is serious when the non-cooperative game defined by $h^{\alpha CP(\mathbf{J})}$ and

$(e, \mathbf{c}, \mathbf{v})$ has multiple equilibrium outcomes. The reason is that if $T\varepsilon(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})}) \setminus \mathcal{Z}\varepsilon(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})})$ is non-empty, then individuals may realize an outcome in $T\varepsilon(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})}) \setminus \mathcal{Z}\varepsilon(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})})$ through the non-cooperative game $(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})})$. In such a case, the aim of the Rawlsian difference principle cannot be realized. In this section, we propose a resolution of this "moral hazard" problem.

Assume a pure-strategy Nash equilibrium concept. Given $\mathbf{J} \in \mathcal{J}^n$ and $\alpha \in [0,1]$, let $NE^{-\mathcal{Z}}(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})}) := \{\mathbf{x} \in NE(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})}) \mid (\overline{x} - x_i, h_i^{\alpha CP(\mathbf{J})}(e, \mathbf{c}, \mathbf{x}))_{i \in N} \in NA(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})}) \setminus \mathcal{Z}(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})})\}$ and $NE^{\mathcal{Z}}(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})}) := \{\mathbf{x} \in NE(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})}) \mid (\overline{x} - x_i, h_i^{\alpha CP(\mathbf{J})}(e, \mathbf{c}, \mathbf{x}))_{i \in N} \in \mathcal{Z}(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})})\}$.
Given $\mathbf{J} \in \mathcal{J}^n$, $\alpha \in [0,1]$, and $\mathbf{v} \in V^n$, let $h^{\alpha CP(\mathbf{J},\mathbf{v})} : E \times \mathcal{C}^m \times [0, \overline{x}]^n \to \mathbb{R}^n_+$ be a distribution rule defined as follows:

Rule 1: If $\mathbf{x} \in NE^{\mathcal{Z}}(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})})$, then $h^{\alpha CP(\mathbf{J},\mathbf{v})}(e, \mathbf{c}, \mathbf{x}) = h^{\alpha CP(\mathbf{J})}(e, \mathbf{c}, \mathbf{x})$.

Rule 2: If $\mathbf{x} \notin NE(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})})$ and there is a unique individual $j \in N$ such that for some $x'_j \in [0, \overline{x}]$, $(x'_j, x_{-j}) \in NE^{\mathcal{Z}}(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})})$, then $h^{\alpha CP(\mathbf{J},\mathbf{v})}(e, \mathbf{c}, \mathbf{x}) = h^{\alpha CP(\mathbf{J})}(e, \mathbf{c}, \mathbf{x})$.

Rule 3: If $\mathbf{x} \in NE^{-\mathcal{Z}}(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})})$, then $h^{\alpha CP(\mathbf{J},\mathbf{v})}(e, \mathbf{c}, \mathbf{x}) = \mathbf{0}$.

Rule 4: In any other case of $\mathbf{x} \in [0, \overline{x}]^n$, the following modulo game is played and some individual $j(\mathbf{x}) \in N$ will win the game: Let $\sum_N \frac{x_i}{\overline{x}} = p$. Clearly, $0 \leq p \leq n$. Let $r + \nu = p$ where r is the largest integer less than or equal to p. Then, $\nu \in [0,1]$, and there is a unique $j(\mathbf{x}) \in N$ such that $\nu \in [\frac{i^*-1}{n}, \frac{i^*}{n})$. Then, $j(\mathbf{x})$ is able to receive $h_{j(\mathbf{x})}^{\alpha CP(\mathbf{J},\mathbf{v})}(e, \mathbf{c}, \mathbf{x}) = f(\sum_N s_h x_h)$, and for any $i \neq j(\mathbf{x})$, $h_i^{\alpha CP(\mathbf{J},\mathbf{v})}(e, \mathbf{c}, \mathbf{x}) = 0$.

We call such a distribution rule *a* $\mathbf{v}$*-based quasi-*α*-combination rule.* Given $\mathbf{J} \in \mathcal{J}^n$ and $\alpha \in [0,1]$, let $q\alpha CP(\mathbf{J}) := \{h^{\alpha CP(\mathbf{J},\mathbf{v})} \mid \mathbf{v} \in V^n\}$ be the set of $\mathbf{v}$-based quasi-α-combination rules. Let $QCP(\mathbf{J}) := \bigcup_{\alpha \in [0,1]} q\alpha CP(\mathbf{J})$ and $QCP := \bigcup_{\mathbf{J} \in \mathcal{J}^n} QCP(\mathbf{J})$.

Theorem 2: *For each* $\mathbf{v} \in V^n$, $NE^{\mathcal{Z}}(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})}) = NE(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J},\mathbf{v})})$.

Proof. See Appendix.

Consider placing the following condition on $\mathbf{Q}(\cdot,\cdot,\cdot,\cdot) \in \mathcal{Q}^n$:

Second Priority of Quasi-α-Combination Rules (SPQCP): *For every individual $i \in N$, any $\mathfrak{g} \in QCP$, and any $\mathfrak{g}' \in \mathbf{\Gamma}_{CO} \setminus QCP$,*
$[((\mathbf{z},\mathfrak{g}),(\mathbf{z}',\mathfrak{g}')) \in Q_i(\cdot,\cdot,\cdot,\cdot)\&((\mathbf{z}',\mathfrak{g}'),(\mathbf{z},\mathfrak{g})) \notin Q_i(\cdot,\cdot,\cdot,\cdot)$ for all $\mathbf{z}$ and $\mathbf{z}'$ in $Z(E)]$.

Denote the class of judgement correspondences satisfying the conditions of **FPCM**, **SPQCP**, **PFP**, **PEO**, and **CCC** by $\mathcal{Q}_{R^*}$. By replacing **SPCP** with **SPQCP**, we obtain the following *"revised" Rawlsian social decision procedure of fair allocation rules*:

Definition 8: *The* revised Rawlsian social decision procedure of fair allocation rules (**R*SDPR**) *is a function $\Psi_{R^*} : \mathcal{Q}^n_{R^*} \to \mathcal{Q}_{R^*}$ such that for every $\mathbf{Q}(\cdot,\cdot,\cdot,\cdot) = (Q_i(\cdot,\cdot,\cdot,\cdot))_{i\in N} \in \mathcal{Q}^n_{R^*}$, $\Psi_{R^*}(\mathbf{Q}(\cdot,\cdot,\cdot,\cdot)) = Q(\cdot,\cdot,\cdot,\cdot)$, and Ψ_{R^*} satisfies the* Pareto Principle.

By adopting the **R*SDPR** Ψ_{R^*}, we can resolve the "moral hazard" problem discussed above. First, by **SPQCP** and **PFP**, if neither $(\mathbf{z},\mathfrak{g}) \notin Z(E) \times QCP$ nor $\mathfrak{g}(e,\mathbf{c},\mathbf{x}) \neq \mathbf{z}$, then $(\mathbf{z},\mathfrak{g})$ is rejected by $B_{\Psi_R(\cdot)}$. Thus, for each $(e,\mathbf{c}) \in E \times \mathcal{C}^m$, $\mathcal{FP}(e,\mathbf{c}) = \{(\mathbf{z},\mathfrak{g}) \in Z(E) \times QCP \mid \mathfrak{g}(e,\mathbf{c},\mathbf{x}) = \mathbf{z}\}$. Second, by **PEO**, for each $(e,\mathbf{c}) \in E \times \mathcal{C}^m$, each $\mathbf{v} \in V^n$, each $\mathbf{J} \in \mathcal{J}^n$, and each $(\mathbf{z}, h^{\alpha CP(\mathbf{J},\mathbf{v})}) \in \mathcal{FP}(e,\mathbf{c})$, if $\mathbf{z} \notin NA(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})$, then $(\mathbf{z}, h^{\alpha CP(\mathbf{J},\mathbf{v})})$ is rejected by $B_{\Psi_R(\cdot)}$, since $\mathbf{z} \notin NA(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J},\mathbf{v})})$. Moreover, if $\mathbf{z} \notin \mathcal{Z}(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})$, then $(\mathbf{z}, h^{\alpha CP(\mathbf{J},\mathbf{v})})$ is rejected by $B_{\Psi_R(\cdot)}$, since $\mathbf{z} \notin NA(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J},\mathbf{v})})$. Finally, through the test of **CCC**, we obtain that for each $(e,\mathbf{c}) \in E \times \mathcal{C}^m$, each $\mathbf{v} \in V^n$, and each $\mathbf{J} \in \mathcal{J}^n$, $B_{\Psi_{R^*}(\mathbf{Q})}(e,\mathbf{c},\mathbf{v},\mathbf{J}) = B_{\Psi_R(\mathbf{Q})}(e,\mathbf{c},\mathbf{v},\mathbf{J})$. Thus, if individuals' judgements on extended alternatives are $\mathbf{Q}(\cdot,\cdot,\cdot,\cdot) \in \mathcal{Q}^n_R$, and their judgements on common capabilities are $\mathbf{J} \in \mathcal{J}^n$, then, in each economic environment $(e,\mathbf{c},\mathbf{v}) \in E \times \mathcal{C}^m \times V^n$, some quasi-$\alpha$-combination rule $h^{\alpha CP(\mathbf{J},\mathbf{v})}$ is selected through $\Psi_{R^*}(\mathbf{Q}(\cdot,\cdot,\cdot,\cdot))$. In such a case, one of the allocations in $\mathcal{Z}(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})$ is surely realized as a Nash equilibrium outcome of the non-cooperative game $(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J},\mathbf{v})})$, since $NE^{\mathcal{Z}}(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})}) = NE(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J},\mathbf{v})})$. Thus, the aim of the Rawlsian difference principle is realized, and the "moral hazard" problem is resolved.

7 Concluding Remarks

In this paper, we discussed social choice procedure of fair allocation rules from the viewpoint of the Rawlsian principles of justice. As a formulation of this procedure, we propose the **RSDPR**, and show that this function is well-defined under some assumptions. Moreover, we discuss a "moral hazard" problem in implementing the aim of the difference principle by the **RSDPR**.

There is still a selection problem of fair allocation rules in each economic environment, although we did not discuss it in this paper. By **PEO** and **CCC**, the selection problem of fair allocation rules involves the prediction about what resource allocations are realized under each candidate of fair allocation rules. Hence, the selection of a fair allocation rule in each economic environment depends upon the information regarding the characteristics the economy has. In each economic environment $(e, \mathbf{c}, \mathbf{v}) \in E \times \mathcal{C}^m \times V^n$, the selection of an appropriate α-combination rule $h^{\alpha CP(\mathbf{J})}$ such that $\alpha \in \mathcal{A}\varepsilon(e, \mathbf{c}, \mathbf{v}, CP(\mathbf{J}))$ depends upon the information of $(e, \mathbf{c}, \mathbf{v})$. This dependency implies that even if $(e, \mathbf{c})$ is observable, each individual i can manipulate the selection process of $h^{\alpha CP(\mathbf{J})}$ by "false-telling" her private information v_i. As a result, the selection of the fair allocation rule may be incorrect in this economy, so that the aim of the difference principle may not be implemented. This is a problem of adverse selection. Gotoh and Yoshihara (1998) discuss this problem, and try to resolve it by using implementation theory.

Note that in this paper, we do not analytically discuss the problem of why and how the society agrees to accept the Rawlsian two principles of justice. It is an important further issue which we should try to analyze.

Appendix

Proof of Proposition 1: Let $(e, \mathbf{c}, \mathbf{v}) \in E^c \times \mathcal{C}^m \times V^n$ where $e = (\mathbf{a}, \mathbf{s}, f)$. Since f is continuous and concave, it is easy to verify that h_i^{PR} is continuous and concave. Then, the payoff function of the non-cooperative game $(e, \mathbf{c}, \mathbf{v}, h^{PR})$, $u_i^{h^{PR}} : [0, \overline{x}]^n \times \mathbf{C}_e^h([0, \overline{x}]^n) \to \mathbb{R}$ is continuous on $[0, \overline{x}]^n$. This is followed by the continuity of v_i and the Berge's maximum theorem.

We next show that for any given $x_{-i} \in [0, \overline{x}]^{n-1}$, the payoff function $u_i^{h^{PR}}(\cdot, x_{-i})$ is also quasi-concave on $[0, \overline{x}]$. Given $x_i, x_i' \in [0, \overline{x}]$ such that $x_i' \neq x_i$, let $b_i \in \partial C(a_i, \overline{x} - x_i, h_i^{PR}(x_i, x_{-i}))$ and $b_i' \in \partial C(a_i, \overline{x} - x_i', h_i^{PR}(x_i', x_{-i}))$.

Note that for b_i (resp. b'_i), there are $\boldsymbol{\beta} = (\beta_k)_{k\in\{1,\cdots,m\}}$ and $\boldsymbol{\gamma} = (\gamma_k)_{k\in\{1,\cdots,m\}}$ (resp. $\boldsymbol{\beta}' = (\beta'_k)_{k\in\{1,\cdots,m\}}$ and $\boldsymbol{\gamma}' = (\gamma'_k)_{k\in\{1,\cdots,m\}}$) such that $\sum_{k=1}^{m} \beta_k = 1$, $\sum_{k=1}^{m} \gamma_k = 1$, and $c_k(a_i, \beta_k \cdot (\overline{x}-x_i), \gamma_k \cdot h_i^{PR}(x_i, x_{-i})) = b_{ik}$ for every functioning $k \in \{1,\cdots,m\}$. Given $\lambda \in [0,1]$, let $x_i(\lambda) := \lambda x_i + (1-\lambda)x'_i$.

Note that $\sum_{k=1}^{m} \{\lambda\beta_k \cdot (\overline{x} - x_i) + (1-\lambda)\beta'_k \cdot (\overline{x} - x'_i)\} = \overline{x} - x_i(\lambda)$. Let $\beta_k \cdot (\overline{x}-x_i) = X_k$ and $\beta'_k \cdot (\overline{x}-x'_i) = X'_k$. Then, $\lambda\beta_k \cdot (\overline{x}-x_i) + (1-\lambda)\beta'_k \cdot (\overline{x}-x'_i) = \lambda X_k + (1-\lambda)X'_k = X_k(\lambda)$. Since $\sum_{k=1}^{m} X_k(\lambda) = \overline{x} - x_i(\lambda)$, there exists $\boldsymbol{\beta}(\lambda) = (\beta_k(\lambda))_{k\in\{1,\cdots,m\}}$ such that $\sum_{k=1}^{m} \beta_k(\lambda) = 1$ and for every functioning $k \in \{1,\cdots,m\}$, $\beta_k(\lambda) \cdot (\overline{x} - x_i(\lambda)) = X_k(\lambda)$.

Note that $\sum_{k=1}^{m} \{\lambda\gamma_k \cdot h_i^{PR}(x_i, x_{-i}) + (1-\lambda)\gamma'_k \cdot h_i^{PR}(x'_i, x_{-i})\}$ $= \lambda h_i^{PR}(x_i, x_{-i}) + (1-\lambda) \cdot h_i^{PR}(x'_i, x_{-i}) \leq h_i^{PR}(x_i(\lambda), x_{-i})$ by concavity of $h_i^{PR}(\cdot, x_{-i})$. Let $\gamma_k \cdot h_i^{PR}(x_i, x_{-i}) = Y_k$ and $\gamma'_k \cdot h_i^{PR}(x'_i, x_{-i}) = Y'_k$. Then, $\lambda\gamma_k \cdot h_i^{PR}(x_i, x_{-i}) + (1-\lambda)\gamma'_k \cdot h_i^{PR}(x'_i, x_{-i}) = \lambda Y_k + (1-\lambda)Y'_k = Y_k(\lambda)$. Since $\sum_{k=1}^{m} Y_k(\lambda) \leq h_i^{PR}(x_i(\lambda), x_{-i})$, there exists $\boldsymbol{\gamma}(\lambda) = (\gamma_k(\lambda))_{k\in\{1,\cdots,m\}}$ such that $\sum_{k=1}^{m} \gamma_k(\lambda) = 1$ and for every functioning $k \in \{1,\cdots,m\}$, $\gamma_k(\lambda) \cdot h_i^{PR}(x_i(\lambda), x_{-i}) \geq Y_k(\lambda)$.

Since c_k is concave, for every functioning $k \in \{1,\cdots,m\}$,

$$\begin{gathered}\lambda b_{ik} + (1-\lambda)b'_{ik} = \\ \lambda \cdot c_k(a_i, \beta_k \cdot (\overline{x}-x_i), \gamma_k \cdot h_i^{PR}(x_i, x_{-i})) + (1-\lambda) \cdot c_k(a_i, \beta'_k \cdot (\overline{x}-x'_i), \gamma'_k \cdot h_i^{PR}(x'_i, x_{-i})) \\ \leq c_k(a_i, \beta_k(\lambda) \cdot (\overline{x} - x_i(\lambda)), Y_k(\lambda)) \\ \leq c_k(a_i, \beta_k(\lambda) \cdot (\overline{x} - x_i(\lambda)), \gamma_k(\lambda) \cdot h_i^{PR}(x_i(\lambda), x_{-i})).\end{gathered}$$

This implies that $\lambda b_i + (1-\lambda)b'_i \in C(a_i, \overline{x} - x_i(\lambda), h_i^{PR}(x_i(\lambda), x_{-i}))$. Thus, $C(a_i, \cdot, h_i^{PR}(\cdot, x_{-i}))$ has a convex graph. Therefore, we conclude that $u_i^{h^{PR}}(\cdot, x_{-i})$ is quasi-concave on $[0, \overline{x}]$.

Thus, by the theorem of Nash (1951), $NA(e, \mathbf{c}, \mathbf{v}, h^{PR}) \neq \varnothing$. ■

The following two lemmas are used in order to prove Theorem 1.

Lemma 1: *If h is continuous, then $NA(e, \mathbf{c}, \mathbf{v}, h)$ is compact.*

Proof. It is enough to show the statement in the case of $NA(e, \mathbf{c}, \mathbf{v}, h) \neq \varnothing$. To show this, we use the following proposition:

Proposition 3 (Border (1985;12.9)): *Let $K \subset \mathbb{R}^m$ be compact, $G \subset \mathbb{R}^n$, and let $\gamma : K \times G \twoheadrightarrow K$ be closed. Put $F(g) = \{x \in K \mid x \in \gamma(x, g)\}$. Then $F : G \twoheadrightarrow K$ has compact values, is closed and upper hemi-continuous.*

Given $i \in N$, let $G_i = NE_{-i}(e, \mathbf{c}, \mathbf{v}, h)$. Let $r_i : [0, \overline{x}]^n \twoheadrightarrow [0, \overline{x}]$ be the best-response correspondence of i such that for all $\mathbf{x} \in [0, \overline{x}]^n$, $r_i(\mathbf{x}) = \{x_i^* \in [0, \overline{x}] \mid u_i^h(x_i^*, x_{-i},) \geq u_i^h(x_i', x_{-i},)$ for all $x_i' \in [0, \overline{x}]\}$. Since h is continuous, we can verify that $r_i(\mathbf{x})$ is closed correspondence. Let $\gamma_i : [0, \overline{x}] \times G \twoheadrightarrow [0, \overline{x}]$ be such that for all $\mathbf{x} \in [0, \overline{x}] \times G$, $\gamma_i(\mathbf{x}) = r_i(\mathbf{x})$. Then, clearly, γ_i is closed correspondence. For each $x_{-i} \in NE_{-i}(e, \mathbf{c}, \mathbf{v}, h)$, let $F_i(x_{-i}) = \{x_i \in [0, \overline{x}] \mid x_i \in \gamma_i(x_i, x_{-i})\}$. Then, by the above proposition, $F_i : NE_{-i}(e, \mathbf{c}, \mathbf{v}, h) \twoheadrightarrow [0, \overline{x}]$ has compact values and closed correspondence. Let $F : NE(e, \mathbf{c}, \mathbf{v}, h) \twoheadrightarrow [0, \overline{x}]^n$ be such that for all $\mathbf{x} \in NE(e, \mathbf{c}, \mathbf{v}, h)$, $F(\mathbf{x}) = \underset{i \in N}{\times} F_i(x_{-i})$. Thus, F is closed correspondence. Since $F(NE(e, \mathbf{c}, \mathbf{v}, h)) = NE(e, \mathbf{c}, \mathbf{v}, h)$, this implies $NE(e, \mathbf{c}, \mathbf{v}, h)$ is closed subset of $[0, \overline{x}]^n$, so that $NE(e, \mathbf{c}, \mathbf{v}, h)$ is compact. ■

Lemma 2: *For all $\alpha \in [0, 1]$, all $e = (\mathbf{a}, \mathbf{s}, f) \in E$, all $\mathbf{c} \in \mathcal{C}^m$, and all $\mathbf{v} \in V^n$, if $h^{\alpha CP(\mathbf{J})}$ and $\psi(\mathbf{J})$ are continuous, and $NA(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})}) \neq \varnothing$, then $\mathcal{Z}(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})})$ is non-empty and compact.*

Proof. Given $e = (\mathbf{a}, \mathbf{s}, f) \in E$, let $Y_e : [0, \overline{x}]^n \twoheadrightarrow \mathbb{R}_+^n$ be such that $Y_e(\mathbf{x}) = Y(\mathbf{s}, f, \mathbf{x})$. By Gotoh and Yoshihara (1997), if $\psi(\mathbf{J})$ is continuous, we can construct a continuous ordering $\mathcal{R}_e(\psi(\mathbf{J})) \subseteq ([0, \overline{x}]^n \times Y_e([0, \overline{x}]^n))^2$ such that $(\mathbf{z}, \mathbf{z}') \in \mathcal{R}_e(\psi(\mathbf{J}))$ if and only if $(CC(e, \mathbf{c}, \mathbf{z}), CC(e, \mathbf{c}, \mathbf{z}')) \in \psi(\mathbf{J})$. Then, we can construct a continuous function $w_e^{\psi(\mathbf{J})} : [0, \overline{x}]^n \times Y_e([0, \overline{x}]^n) \to \mathbb{R}$ which represents $\mathcal{R}_e(\psi(\mathbf{J}))$. Define $w^{\psi(\mathbf{J})}_{NA(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})} : NA(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})}) \to \mathbb{R}$ such that for all $\mathbf{z} \in NA(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})})$, $w^{\psi(\mathbf{J})}_{NA(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})}(\mathbf{z}) = w_e^{\psi(\mathbf{J})}(\mathbf{z})$. Since $h^{\alpha CP(\mathbf{J})}$ is continuous, by Lemma 1 and the Weierstrass Theorem, $\mathcal{Z}(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})})$ is non-empty and compact. ■

Proof of Theorem 1: By Gotoh and Yoshihara (1997), under Assumption 1, if $\psi(\mathbf{J})$ is continuous, then $h^{CM(\mathbf{J})}$ is continuous. Thus, for all $e = (\mathbf{a}, \mathbf{s}, f) \in$

E, all $\mathbf{c} \in \mathcal{C}^m$, and all $\alpha \in [0,1]$, $h^{\alpha CP(\mathbf{J})}(e,\mathbf{c},\cdot)$ is continuous on $[0,\overline{x}]^n$. Without loss of generality, let us assume that $(e,\mathbf{c},\mathbf{v}) \in NA^{-1}(h^{CM(\mathbf{J})})$. Hence, $NA(e,\mathbf{c},\mathbf{v},h^{CM(\mathbf{J})}) \neq \varnothing$. Thus, by Lemma 2, $\mathcal{Z}(e,\mathbf{c},\mathbf{v},h^{CM(\mathbf{J})})$ is non-empty and compact. For each $\alpha \in [0,1]$, let us define $\mathcal{QZ}(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})$ as follows:

$$\mathcal{QZ}(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})}) := \begin{cases} \mathcal{Z}(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})}) & \text{if } NA(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})}) \neq \varnothing \\ \mathcal{Z}(e,\mathbf{c},\mathbf{v},h^{\alpha^*(\alpha) CP(\mathbf{J})}) & \text{if } NA(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})}) = \varnothing \end{cases}$$

where $\alpha^*(\alpha) = \max\{\alpha' \in [0,\alpha] \mid NA(e,\mathbf{c},\mathbf{v},h^{\alpha' CP(\mathbf{J})}) \neq \varnothing\}$.

By this construction, $\bigcup_{\alpha\in[0,1]} \mathcal{QZ}(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})}) = \bigcup_{\alpha\in[0,1]} \mathcal{Z}(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})$. Since by Lemma 2, $\bigcup_{\alpha\in[0,1]} \mathcal{QZ}(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})$ is non-empty, so is $\bigcup_{\alpha\in[0,1]} \mathcal{Z}(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})$. In the following, we also show that $\bigcup_{\alpha\in[0,1]} \mathcal{QZ}(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})$ is compact.

Let $\gamma : Z(e) \times [0,1] \twoheadrightarrow Z(e)$ be such that for any $\mathbf{z} \in Z(e)$ and $\alpha \in [0,1]$, $\gamma(\mathbf{z},\alpha) = \{\mathbf{z}^* \in [0,\overline{x}]^n \mid \mathbf{z}^* = \arg \min_{\mathbf{z}' \in \mathcal{QZ}(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})} \| \mathbf{z}, \mathbf{z}' \| \}$. Since $\mathcal{QZ}(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})$ is closed by Lemma 2, by definition of γ, it is clear that γ is closed correspondence. Let $F(\alpha) = \{\mathbf{z} \in Z(e) \mid \mathbf{z} \in \gamma(\mathbf{z},\alpha)\}$. Then, by Proposition 3, $F : [0,1] \twoheadrightarrow Z(e)$ is compact-valued and upper hemi-continuous, and $F([0,1]) = \bigcup_{\alpha\in[0,1]} \mathcal{QZ}(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})$. Thus, $\bigcup_{\alpha\in[0,1]} \mathcal{QZ}(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})$ is compact, so is $\bigcup_{\alpha\in[0,1]} \mathcal{Z}(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})$.

Define a continuous function $w^{\psi(\mathbf{J})}_{\mathcal{Z}(e,\mathbf{c},\mathbf{v},h^{CP(\mathbf{J})})} : \bigcup_{\alpha\in[0,1]} \mathcal{Z}(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})}) \to \mathbb{R}$ such that for all $\mathbf{z} \in \bigcup_{\alpha\in[0,1]} \mathcal{Z}(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})})$, $w^{\psi(\mathbf{J})}_{\mathcal{Z}(e,\mathbf{c},\mathbf{v},h^{CP(\mathbf{J})})}(\mathbf{z}) = w^{\psi(\mathbf{J})}_e(\mathbf{z})$, which represents $\psi(\mathbf{J})$ over $\mathcal{CC}(\bigcup_{\alpha\in[0,1]} \mathcal{Z}(e,\mathbf{c},\mathbf{v},h^{\alpha CP(\mathbf{J})}))$. By the Weierstrass Theorem, $\mathcal{A}(e,\mathbf{c},\mathbf{v},CP(\mathbf{J}))$ is non-empty. This implies that $S^{\mathcal{A}(CP(\mathbf{J}))}(e,\mathbf{c},\mathbf{v})$ is non-empty. ■

Proof of Corollary 1: By the construction of $\Psi_R(\cdot)$, for any $(e,\mathbf{c},\mathbf{v},\mathbf{J}) \in E \times \mathcal{C}^m \times V^n \times \mathcal{J}^n$ and any $\mathbf{Q} \in \mathcal{Q}^n_R$, $(\mathbf{z}, h^{\alpha CP(\mathbf{J})}) \in B_{\Psi_R(\mathbf{Q})}(e,\mathbf{c},\mathbf{v},\mathbf{J})$ if and only if $\mathbf{z} \in S^{\mathcal{A}(CP(\mathbf{J}))}(e,\mathbf{c},\mathbf{v})$ and $h^{\alpha CP(\mathbf{J})} \in D_{\Psi_R(\mathbf{Q})}(e,\mathbf{c},\mathbf{v},\mathbf{J})$. By Theorem 1, for any $(e,\mathbf{c},\mathbf{v}) \in NA^{-1}(h^{CM(\mathbf{J})}) \cup NA^{-1}(h^{PR})$ and any continuous ordering $\psi(\mathbf{J})$, $S^{\mathcal{A}(CP(\mathbf{J}))}(e,\mathbf{c},\mathbf{v})$ is non-empty. For each $\mathbf{z} \in S^{\mathcal{A}(CP(\mathbf{J}))}(e,\mathbf{c},\mathbf{v})$ and $\mathbf{Q} \in \mathcal{Q}^n_R$, there is $\alpha \in \mathcal{A}(e,\mathbf{c},\mathbf{v},CP(\mathbf{J}))$ such that $(\mathbf{z}, h^{\alpha CP(\mathbf{J})}) \in B_{\Psi_R(\mathbf{Q})}(e,\mathbf{c},\mathbf{v},\mathbf{J})$. By definition, $h^{\alpha CP(\mathbf{J})} \in D_{\Psi_R(\mathbf{Q})}(e,\mathbf{c},\mathbf{v},\mathbf{J})$. ■

Proof of Theorem 2: First, we show that for each $\mathbf{v} \in V^n$, $NE^{\mathbf{Z}}(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})}) \subseteq NE(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J},\mathbf{v})})$. Let $\mathbf{x} \in NE^{\mathbf{Z}}(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})})$. By Rule 1 of $h^{\alpha CP(\mathbf{J},\mathbf{v})}$, $h^{\alpha CP(\mathbf{J},\mathbf{v})}(e, \mathbf{c}, \mathbf{x}) = h^{\alpha CP(\mathbf{J})}(e, \mathbf{c}, \mathbf{x})$. By Rule 2 of $h^{\alpha CP(\mathbf{J},\mathbf{v})}$, for any $j \in N$, any $x_j'(\neq x_j) \in [0, \overline{x}]$, $h^{\alpha CP(\mathbf{J},\mathbf{v})}(e, \mathbf{c}, x_j', x_{-j}) = h^{\alpha CP(\mathbf{J})}(e, \mathbf{c}, x_j', x_{-j})$. Since by $\mathbf{x} \in NE^{\mathbf{Z}}(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})})$, $v_j(b_j(e, \mathbf{c}, \mathbf{v}, \mathbf{x}, h^{\alpha CP(\mathbf{J})})) \geq v_j(b_j(e, \mathbf{c}, \mathbf{v}, x_j', x_{-j}, h^{\alpha CP(\mathbf{J})}))$, we obtain that $v_j(b_j(e, \mathbf{c}, \mathbf{v}, \mathbf{x}, h^{\alpha CP(\mathbf{J},\mathbf{v})})) \geq v_j(b_j(e, \mathbf{c}, \mathbf{v}, x_j', x_{-j}, h^{\alpha CP(\mathbf{J},\mathbf{v})}))$. This implies that $\mathbf{x} \in NE^{\mathbf{Z}}(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J},\mathbf{v})})$.

Second, we show that for each $\mathbf{v} \in V^n$, $NE^{\mathbf{Z}}(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})}) \supseteq NE(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J},\mathbf{v})})$. Let $\mathbf{x} \in NE^{\mathbf{Z}}(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J},\mathbf{v})})$. It is easy to see that $\mathbf{x}$ does not correspond to Rule 3 or 4 of $h^{\alpha CP(\mathbf{J},\mathbf{v})}$. Suppose that $\mathbf{x}$ corresponds to Rule 2 of $h^{\alpha CP(\mathbf{J},\mathbf{v})}$. Then, $\mathbf{x} \notin NE(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})})$ and $h^{\alpha CP(\mathbf{J},\mathbf{v})}(e, \mathbf{c}, \mathbf{x}) = h^{\alpha CP(\mathbf{J})}(e, \mathbf{c}, \mathbf{x})$. Moreover, there exists $j \in N$ such that for some $x_j'(\neq x_j) \in [0, \overline{x}]$, $(x_j', x_{-j}) \in NE^{\mathbf{Z}}(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})})$. Thus, for all $i \in N$, $v_i(b_i(e, \mathbf{c}, \mathbf{v}, x_j', x_{-j}, h^{\alpha CP(\mathbf{J})})) \geq v_i(b_i(e, \mathbf{c}, \mathbf{v}, \mathbf{x}, h^{\alpha CP(\mathbf{J})}))$. By Rule 1 of $h^{\alpha CP(\mathbf{J},\mathbf{v})}$, $h^{\alpha CP(\mathbf{J},\mathbf{v})}(e, \mathbf{c}, x_j', x_{-j}) = h^{\alpha CP(\mathbf{J})}(e, \mathbf{c}, x_j', x_{-j})$. This implies that for all $i \in N$, $v_i(b_i(e, \mathbf{c}, \mathbf{v}, x_j', x_{-j}, h^{\alpha CP(\mathbf{J},\mathbf{v})})) \geq v_i(b_i(e, \mathbf{c}, \mathbf{v}, \mathbf{x}, h^{\alpha CP(\mathbf{J},\mathbf{v})}))$. Since $\mathbf{x} \in NE(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J},\mathbf{v})})$, $v_i(b_i(e, \mathbf{c}, \mathbf{v}, x_j', x_{-j}, h^{\alpha CP(\mathbf{J},\mathbf{v})})) = v_i(b_i(e, \mathbf{c}, \mathbf{v}, \mathbf{x}, h^{\alpha CP(\mathbf{J},\mathbf{v})}))$ for all $i \in N$. This implies $\mathbf{x} \in NE(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})})$, so that a contradiction. Thus, $\mathbf{x}$ does not correspond to Rule 2 of $h^{\alpha CP(\mathbf{J},\mathbf{v})}$. Suppose that $\mathbf{x}$ corresponds to Rule 1 of $h^{\alpha CP(\mathbf{J},\mathbf{v})}$. Then, by definition of $h^{\alpha CP(\mathbf{J},\mathbf{v})}$, $h^{\alpha CP(\mathbf{J},\mathbf{v})}(e, \mathbf{c}, \mathbf{x}) = h^{\alpha CP(\mathbf{J})}(e, \mathbf{c}, \mathbf{x})$. Moreover, for any $j \in N$, any $x_j'(\neq x_j) \in [0, \overline{x}]$, by Rule 2 of $h^{\alpha CP(\mathbf{J},\mathbf{v})}$, $h^{\alpha CP(\mathbf{J},\mathbf{v})}(e, \mathbf{c}, x_j', x_{-j}) = h^{\alpha CP(\mathbf{J})}(e, \mathbf{c}, x_j', x_{-j})$. Since $\mathbf{x} \in NE(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J},\mathbf{v})})$, we obtain that $\mathbf{x} \in NE(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})})$. Thus, $NE^{\mathbf{Z}}(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J})}) \supseteq NE(e, \mathbf{c}, \mathbf{v}, h^{\alpha CP(\mathbf{J},\mathbf{v})})$.
■

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